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Signaling and Optimal Sorting

Tim Perri
Appalachian State University

Department of Economics
Appalachian State University
Boone, NC 28608
Phone: (828) 262-2148
Fax: (828) 262-6105
www.business.appstate.edu/economics

Signaling and Optimal Sorting

by

Timothy Perri[*]

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Abstract

I consider education as a discrete signal of inherent ability. More able individuals are more productive in the *primary sector*, and less able individuals are more productive in the *secondary sector*. If, absent signaling, all would be in the secondary sector, signaling *increases* but never *maximizes* welfare. When all would be in the primary sector without signaling, signaling *may* increase welfare. Interestingly, signaling is more likely to increase welfare the greater is productivity in the secondary sector, and, possibly, the lower is productivity in the primary sector. Excessive signaling occurs by *less able individuals*, which is consistent with recent increased undergraduate enrollment in the U.S. If education increases human capital, total welfare likely increases. However, unless all invest in education, the more human capital is increased by education, the greater the number of individuals who over-invest in education.

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[*] Department of Economics, Appalachian State University, Boone, NC, 28608. E-mail: perritj@gmail.com.
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1. Introduction

Scholars and pundits continue to debate whether education increases productivity (Leonhardt, 2011). In a study for the Social Science Research Council, Arum *et al*. (2011) find that 36% of U.S. college students learn very little after four years. One economist (Bryan Caplan, 2011) recently argued that little is learned in college, higher education is essentially a signal, and welfare would increase if students spent fewer years in school.[1]

One problem for the signaling hypothesis is that potentially cheaper screening mechanisms should exist. Gary Becker (1993) suggested "Surely a year on the job or a systematic interview and applicant-testing program must be a much cheaper and effective way to screen."[2] However, in the European Union, applicant test responses are personal data, and some countries (Greece and France in particular) make employment tests difficult (Perritt, 2008). In the U.S., limits to the use of applicant testing are the U.S. Supreme Court decision *Griggs v. Duke Power* (1971), and subsequent court rulings and legislation. O'Keefe and Vedder (2008) argue that employers now focus on very narrow subjects directly related to jobs in applicant testing. After Griggs, there has been less use of general aptitude and intelligence tests that might help employers determine an individual's ability to learn and advance at a firm. O'Keefe and Vedder suggest that educational signaling has substituted for employment testing since nothing prevents individuals from obtaining educational credentials to try to demonstrate ability to firms.[3]

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[1] Caplan has a forthcoming book, *The Case Against Education*, in which he argues that 80% of the effect of education is signaling.

[2] Becker (1993, p.8).

[3] At least one National Football League team, the Eagles under coach Chip Kelly, asked potential draftees many questions about their college academics (why a player chose his major, what was his hardest class, etc.). The team

Assuming that signaling occurs, I will address two issues. First, who (if anyone) over-invests in education when it is a signal? The typical signaling[4] model (Spence, 1974, and Riley, 2001, for example) implies that more able individuals over-invest in education in order to sort themselves from those who are less able. Although such an argument might explain more individuals receiving, for example, an MBA in order to sort themselves, the recent significant increase in undergraduate enrollment in the U.S. cannot have come from the more able unless relatively few of them previously enrolled in college. From 1995 to 2010, the 18-24 year old population in the U.S. increased from 24.28 million to 27.16 million, a 12% increase.[5] Yet the number enrolled in college in that period rose from 14.26 million to 21.02 million, a 47% increase.[6] As will be demonstrated, this large increase in undergraduate enrollment is consistent with the idea that relatively low ability individuals may over-invest in education in order to distinguish themselves from those with even lower ability.

Second, an additional question is whether signaling may improve welfare by allowing individuals to be allocated to jobs where they are the most productive. Kenneth Arrow was asked about education being wasteful in the signaling model. Arrow responded "It permits the sorting.

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believed that college graduation is more than proof of intelligence, and is a signal that an individual is committed to achieving goals (Clark, 2014).

[4] Some claim evidence of fairly rapid employer learning of skills implies a low value of educational signaling (Altonji and Pierret, 1997, and Lange, 2007). However, Habermalz (2011) shows such a conclusion is unwarranted. Habermalz demonstrates that signaling is not necessary with very rapid employer learning. With very slow employer learning, signaling would not likely be worthwhile due to the short period for which ability would be revealed. With an intermediate length of employer learning, the value of the signal increases with the speed of learning. Estimated returns to education are consistent with theoretical results that imply that signaling occurs. Waldman (2016) combines educational signaling with signaling to other firms by promotion decisions. Waldman argues that studies such as that by Lange (2007) underestimate the returns to signaling because of the assumption such returns are low late in an employee's work life. Waldman finds that educational signaling has potential returns later in individuals' careers when there is asymmetric learning (other firms know less about productivity of a worker than does the worker's employer) and promotion signaling. For simplicity, I ignore different jobs at the same employer, and thus there are no promotion possibilities.

[5] Source: US Bureau of the Census Current Population Reports.

[6] Source: *statista*: http://www.statista.com/statistics/183995/us-college-enrollment-and-projections-in-public-and-private-institutions/

That in itself may be socially gainful. And Spence's model in its form doesn't fully convey all its implications. It implies that an individual will be productive no matter what."[7]

There has been little formal analysis of the sorting effect of educational signaling.[8] Consequently, I develop a theoretical model that avoids the argument about how much education increases human capital/productivity by generally assuming that there is no direct effect of education on productivity. I make such an assumption in order to consider if education can increase welfare by sorting individuals to jobs. Later (Section Six) I allow for the possibility that education increases human capital.

I assume that there may be a social gain from education as a signal if it allows individuals to be sorted to jobs where their productivity is the highest. The usual signaling models generally ignore the sorting gain from education and assume a continuous signal. Such models have either a few types of individuals in terms of ability (typically two) or a continuum of individual ability. In those models, an individual of one ability type may be perfectly identified via signaling. In contrast, I assume a continuum of ability, with individuals getting either zero education (beyond some level assumed to already be attained by all), or one additional level of education. Thus, different ability types attain the same education level---zero or the additional level.

The truth lies somewhere between the usual assumption that one can be perfectly identified and my assumption of many ability types getting the same level of education. The question is which assumption fits the evidence. The number enrolling in college increasing much faster than the school age population suggests that those in the lower ability range of college

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[7] *The Region* (1995, p.8).

[8] Willis (1986) mentions the possibility signaling may be socially worthwhile if the allocation of individuals to jobs matters. For early work that considered sorting, see Spence (1974) and Stiglitz (1975), both of which are discussed in Section Two below.

matriculants try to distinguish themselves from those who are even less able, which is consistent with the assumption of a discrete level of education.[9]

The outline of the rest of this paper is as follows. In Section Two, I compare my approach to what has been done in other signaling models. My model is outlined in Section Three. In Section Four, I consider the case when all would be in the secondary sector if signaling (education) were prohibited, which is the sector in which the productivity of less able individuals is highest. More able individuals are more productive in the primary sector than they are in the secondary sector. In Section Five, I consider the case when all would be in the primary sector absent signaling. The model's implications for technical change and wages are discussed in Section Six. The possibility that education increases human capital is considered in Section Seven, and Section Eight contains a summary.

2. Comparison to previous research

The closest approach to my model is in Lazear (1986). He considers a two-sector model in which productivity is proportional to individual ability in the primary sector, with an alternative that pays the same for all, $\bar{w}$. Information is symmetric: all are uninformed about productivity in the primary sector. If productivity is measured, one is paid a wage equal to productivity minus measurement cost in the primary sector. Measurement of productivity in Lazear's model is equivalent to an accurate test. If measurement is not possible, and expected

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[9] With a continuum of ability, and a continuous signal, all but the least able over-invest in education when it is only a signal and there is no sorting gain from education (Riley, 2001). Such a model requires all to invest more in education if, say, the difference in education costs between ability levels is reduced. It is hard to reconcile that result with the large recent increase in enrollment in the U.S., which suggests that only some individuals have increased the level of their educational signal.

productivity exceeds $\overline{w}$, all work in salary firms in the primary sector. However, those with productivity less than $\overline{w}$ should work elsewhere. Sorting via measurement cost achieves a higher output but at a cost. Lazear considers the asymmetric information case---primary sector employers do not know individual ability, but individuals know their own ability---but not with an alternative sector. Then there is no possible social gain from testing/signaling since there is no sorting.

Spence (2002) considers a model based on Lazear (1986) when there is asymmetric information and no alternative sector. Productivity is learned at a cost (the same for all). Thus, in contrast to his basic model (Spence, 1974), the signal is not a continuous variable, and it directly reveals ability. The net benefit of the signal/test is positively related to productivity, which is necessary for a signaling equilibrium since cost is the same for all. Separating and pooling equilibria exist simultaneously. As in Lazear (1986), when there is no alternative sector so all individuals have the same productivity at any firm, there is no social gain from signaling.

In Regev (2012), more able individuals have a higher benefit but the same cost as the less able from signaling. Two discrete signals are possible, and equilibrium is in mixed strategies. In her model, a pass fail test also exists, there is no sorting gain from education, and education enhances productivity. In contrast, I assume no additional test, there is a sorting gain from education, and (generally) education does not affect productivity. My results are driven by a semi-separating equilibrium which differs from that in Regev who has the same type choosing different education levels. I have a continuum of types, where the more able types choose the discrete level of education, and less able types choose no education. Expected productivity could be higher in either sector, education does not affect productivity (except in Section Seven), and there is a potential social return to signaling from sorting individuals to the sector in which their productivity is highest as suggested by Arrow (*The Region*, 1995).

Additionally, I differ from Regev (2012) in that I consider a discrete signal that is not a test, and that has a cost that is inversely related to ability. The signal implicitly reveals who has productivity above a certain level.[10] In my model, over-investment in education as a signal is not by the more able. Instead, as in Waldman (2016), too many at lower ability levels signal since all with the same education level get the same wage. This is consistent with the significant recent increase in college enrollment in the U.S. discussed in Section One.

Finally, two early attempts to deal with sorting are Spence (1974) and Stiglitz (1975). In an Appendix, Spence considers two different sectors when there is a potential social gain from sorting individuals to the correct sector. With two types of individuals, Spence finds that welfare depends on whether the more able are more or less productive than the less able in the secondary sector.

Stiglitz considers a model with two types of individuals (call them more and less able), two types of jobs (call them skilled and unskilled), and a cost of screening[11] individuals for ability that is the same for all and is perfectly accurate. Stiglitz finds that, if screening cost is too high, there exists a semi-pooling equilibrium in which a fraction of the more able screen. In this case, *too little screening* occurs if the more able are less productive than the less able in the unskilled job. The reason for too little screening is that the less able subsidize the more able in the unskilled job where the wage equals expected productivity.

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[10] Arcidiacono *et al*. (2010) suggest that college may directly reveal aspects of individual ability to employers. I assume no direct revelation is possible. If direct revelation were possible, education would essentially act as a test as in Lazear (1986), Perri (1994), and Spence (2002).

[11] Typically, when individuals incur cost to identify themselves it is called signaling. Firms incurring such cost is called screening.

In my model, there is a continuum of individual ability, and all have the same productivity in the secondary sector. In Sections Four and Five, I show that welfare in my model depends on the productivity of all in either sector,[12] and in which sector individuals would be employed if signaling were prohibited. I find that too much signaling always occurs. However, signaling may still increase welfare.

3. A two sector model with signaling: setup

I consider a world with two sectors. In the *primary sector*, individual ability, x, affects productivity. One can be think of this as the sector with skilled jobs. In the *secondary sector*, x is less important for productivity than in the primary sector. For simplicity, I assume x has no effect on productivity in the secondary sector. There all have productivity of ω, with $\omega > 0$. Information is asymmetric. An individual knows his value of x. Employers know the distribution of x.

I assume x is continuously distributed on the interval $[0, x_{max}]$, with a density of $f(x)$, and a cumulative density of $F(x)$. Productivity in the primary sector is vx, with $v > 0$. I assume that $vx_{max} > \omega$, so there are individuals with $x > \omega/v$ who are more productive in the primary sector, and those with $x < \omega/v$ who are more productive in the secondary sector. Note that a larger v is

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[12] Waldman (2016) has a model that is similar to mine in that there is one education level, there are two wages (for those with and without education, the former hired in the primary sector, and the latter in the secondary sector in my model), signaling cost is inversely related to ability, and too much signaling occurs by those who are *less able*. Waldman is interested in how signaling affects job assignment when said assignment is a signal to alternative employers of individual ability, and when education is directly productive. In particular, in his model, educational signaling reduces the distortion in the promotion decision when promotion is also a signal. I have no promotions, and education is generally not directly productive. I am concerned with the welfare effects of signaling---which always occurs in my model---which depend on in which sector individuals would be if education were banned or taxed away.

consistent with skill-biased technological change since a larger v implies that individuals are more valuable in the primary sector where more ability means greater productivity.

At some cost, $\mathcal{C}(x)$, an individual can acquire a signal, which can be thought of as a discrete level of education. The signal is *not* an accurate test. As in the standard signaling model (Spence, 1974) it is assumed that $\mathcal{C}(x)$ is inversely related to x.[13] Firms in the primary sector can infer who would optimally acquire education, those with $x \geq x^*$.

Above, I assumed that $vx_{max} > \omega$. A stronger assumption implies that signaling always occurs. With no signaling, all would be in either the primary or secondary sector, depending on where average productivity is higher. With $v\mathrm{E}(x)$ the expected productivity if all were in the primary sector, the wage with no signaling would equal $max(v\mathrm{E}(x),\omega)$. I assume that

$vx_{max} > max(v\mathrm{E}(x),\omega) + \mathcal{C}(x_{max})$. With an additional assumption regarding out-of-equilibrium beliefs (see the last paragraph in this section), some will always signal.

Following Waldman (2016), the timing of the game is as follows. First, each individual's value of x is drawn from $\mathrm{F}(x)$. Second, all individuals simultaneously decide whether to obtain education. Third, firms in the two sectors make wage offers. With some individuals obtaining education, primary sector firms will only make wage offers to those with education, and will pay a wage equal to the expected productivity of those who firms believe would optimally choose to acquire education. Those who do not obtain education will be offered a wage of ω in the

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[13] One criticism of signaling models is the assumption that signaling cost and ability are inversely related (Weis, 1983, and Regev, 2012). I believe it is reasonable to assume that signaling cost is inversely related to x. More able individuals require less effort to obtain a given level of education. Also, less study time required for the more able to obtain education implies lower foregone earnings for them.

secondary sector. Fourth, individuals who choose education accept offers from the primary sector. Others accept employment in the secondary sector.[14]

I consider Perfect Bayesian equilibria where it is also assumed that an action off the equilibrium path is by one with the lowest cost of such an action. This implies signaling will always occur. If no signaling occurs, and all are paid the same wage, one with $x = x_{max}$ will deviate from this equilibrium, signal, and earn vx_{max}, which exceeds earnings with no signaling plus signaling cost for these individuals. Thus, there are always some individuals who will signal.

4. All would be in the secondary sector if signaling were prohibited

If $v\mathrm{E}(x) < \omega$, then, absent signaling, all would be in the secondary sector.[15] In this case, the social gain from signaling is that it leads to employment of some in the primary sector who are more productive there than in the secondary sector.

Proposition One. When all would be in the secondary sector if signaling were prohibited, signaling always increases but never maximizes welfare.

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[14] Suppose individuals incorrectly estimate the number who would signal, and thereby understate or overstate what the wage would be in the primary sector. Depending on the direction of the estimation error, welfare could be higher or lower than with no error. For analysis of when individuals misjudge what wages or prices will be, see Akerlof and Tong (2013), Akerlof and Shiller (2015), and Perri (2016).

[15] Bickhchandani, Hirshleifer, and Riley (2013) consider a model with a primary job and an outside opportunity that pays more than the expected productivity of individuals in the primary job. They assume two types of individuals and a continuous signal, so, as is usually the case, any excessive investment in education is by the more able. I consider a continuum of types and a discrete signal.

Proof. Those who signal will be hired in the primary sector and paid a wage equal to expected productivity of those who would find it optimal to signal, those with $x \geq x^*$. Thus, x^* is determined by:

$$vE(x|x \geq x^*) - \mathcal{C}(x^*) = \omega. \tag{1}$$

Since the productivity of the marginal individual who signals is vx^*, the private return to signaling, $vE(x|x \geq x^*)$, exceeds the social return, vx^*, so too much signaling occurs. Those with $x \geq x^*$ signal. The number who do not signal is $F(x^*)$. Then welfare with signaling is:

$$v \int_{x*}^{x_{max}} x\, f(x)dx + \omega F(x^*) - \int_{x*}^{x_{max}} \mathcal{C}(x)f(x)dx, \tag{2}$$

and welfare without signaling is ω. With $E(x|x \geq x^*) = \left(\frac{1}{1-F(x*)}\right) \int_{x*}^{x_{max}} x\, f(x)dx$, and $E(\mathcal{C}|x \geq x^*) = \left(\frac{1}{1-F(x*)}\right) \int_{x*}^{x_{max}} \mathcal{C}(x)f(x)dx$, signaling increases welfare if:

$$[1- F(x^*)][\, vE(x|x \geq x^*) - E(\mathcal{C}|x \geq x^*) - \omega] > 0. \tag{3}$$

Ineq.(3) indicates that signaling increases welfare if the number who signal, $[1- F(x^*)]$, multiplied by the following is positive: for those who signal, the average productivity in the primary sector, minus their average cost of signaling, minus what their output would be in the secondary sector---where all would be employed absent signaling. Using *eq*.(1), *ineq*.(3) becomes:

$$[1- F(x^*)][\mathcal{C}(x^*) - \mathrm{E}(\mathcal{C}|x \geq x^*)] > 0, \qquad (3')$$

which clearly is true since $\mathcal{C}$ is inversely related to x. □

The expected output gain per person from moving individuals from the secondary sector to the primary sector is $v\mathrm{E}(x|x \geq x^*)$ minus ω, and the expected cost is $\mathrm{E}(\mathcal{C}|x \geq x^*)$. There are two factors that drive the results. First, individuals see the expected gain in output, which exceeds that for the marginal individual, which is why too much signaling occurs. Second, the expected output gain equals the cost of signaling for the marginal individual, which exceeds the average cost of signaling. Thus, signaling necessarily increases welfare *if* the alternative to signaling is that all would be employed in the secondary sector.

An increase in ω or a decrease in v decreases the productivity difference between the primary and secondary sectors. Thus, a larger ω or a smaller v implies more to gain from signaling when, absent signaling, all would be employed in the secondary sector. However, in this case, signaling *always* increases welfare. Since an increase in ω or a decrease in v increases the likelihood all would be in the secondary sector absent signaling, this is one reason (see the next section for other reasons) why a larger ω or a smaller v makes it more likely that signaling increases welfare.

Excessive education occurs because individuals are not perfectly sorted via the educational signal. Although there clearly exist more than one level of education---different degrees, quality of schools, majors, and performance---it seems reasonable that there are fewer discernible levels of education[16] than there are types of individuals, and that wages do not fully adjust to reflect all of the observable measures of education.

Since signaling always occurs in my model, it is not possible to know where individuals would be employed absent signaling. Recent technical change (Section Six) implies a larger value for v, thus making it more likely that all would be in the primary sector absent signaling via education (see Section 5). However, the results in this sub-section are still important for the debate on the social value of education when education is primarily a signal. I show that educational signaling *always* increases welfare when prohibiting education would result in all employed in the secondary sector education, even if education does not increase human capital. What happens when all would be in the primary sector absent signaling is addressed in the next section.

5. All would be in the primary sector if signaling were prohibited

If $v\mathrm{E}(x) > \omega$, all would be in the primary sector if signaling were prohibited. Then the potential social benefit from signaling is that it moves some less able individuals from the

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[16] Suppose there are more than one discrete signals whose cost differs by a constant. At least for the case when ability is distributed uniformally, only the cheapest signal will be obtained, and the results will be the same as in this section. See the Appendix for a proof when there are two discrete signals.

primary sector to the secondary sector. I say potential because, depending on the value of x^*, there may not be a social benefit in this case. The social cost is the cost of signaling, borne by those who wish to remain in the primary sector, but the possible social benefit involves the reallocation of the less able to the secondary sector.

In order to derive explicit solutions, I assume x is distributed uniformally on the interval [0,1]. Also, I assume $\mathcal{C}(x) = c(1-x)$, with $c > 0$.[17] With these assumptions, I can solve *eq*.(1) to find the level of x^*:

$$x^* = max\left(0,\ \frac{2(\omega+c)-v}{v+2c}\right). \tag{4}$$

I focus on the situation when $x^* > 0$. There are two cases to consider.

Case 1. Suppose $x^* < \frac{\omega}{v}$. Since those with $x < \frac{\omega}{v}$ are more productive in the secondary sector than in the primary sector, all who should be in the primary sector are there with signaling. Also, those with $x \in [x^*, \frac{\omega}{v}]$ are in the primary sector, and they are less productive there than in the secondary sector. However, I wish to look at the welfare difference, Ω, when signaling is feasible versus when it is not feasible. Since, absent signaling, all would be in the primary sector, those with $x \in [x^*, \frac{\omega}{v}]$ do not affect Ω.

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[17] With $x_{max} = 1$, $\mathcal{C}(1) = 0$. This is not important. I could assume some additional fixed cost of signaling for all. As long as $v - \mathcal{C}(1) > \omega$, some will signal.

The average productivity in the primary sector of those who do not signal is $\frac{vx*}{2}$. Thus, for the x* individuals in the secondary sector, output increases on average by $\left(\omega - \frac{vx*}{2}\right)$, so the total output gain from signaling is then:

$$x^*\left(\omega - \frac{vx*}{2}\right). \tag{5}$$

Case 2. Now suppose $x^* > \frac{\omega}{v}$. Those with $x \in \left[\frac{\omega}{v}, x*\right]$ are more productive in the primary sector than in the secondary sector, but are in the latter sector. The *loss* in output from those who would be in the primary sector *without* signaling, are in the secondary sector *with* signaling, and are more productive in the primary sector is:

$$\int_{\omega/v}^{x*} (vx - \omega)dx,$$

$$\frac{vx^2}{2}\Big|_{\omega/v}^{x*} - \omega\left(x * - \frac{\omega}{v}\right),$$

$$\frac{v}{2}\left[(x*)^2 - \left(\frac{\omega}{v}\right)^2\right] - \omega\left(x * - \frac{\omega}{v}\right),$$

$$\left(x * - \frac{\omega}{v}\right)\left(\frac{vx*-\omega}{2}\right). \tag{6}$$

For the other individuals moved from the primary sector to the secondary sector with signaling, those with $x < \frac{\omega}{v}$, their average productivity in the primary sector would be $\frac{v\left(\frac{\omega}{v}\right)}{2} = \frac{\omega}{2}$. Thus the *gain* from moving them from the primary sector to the secondary sector is:

$$\frac{\omega}{v}\left[\omega - \frac{\omega}{2}\right] = \frac{\omega^2}{2v}. \tag{7}$$

The gain minus the loss from moving individuals from the primary sector to the secondary sector is found by subtracting *eq*.(6) from *eq*.(7), which yields $x^*\left(\omega - \frac{vx*}{2}\right)$---the same term found (*eq*.(5)) when $x^* < \omega/v$. When $vx^* > \omega$, it is possible to have $\frac{vx*}{2} > \omega$, so output actually declines with signaling. However, the expression for the change in output with signaling is the same regardless of whether output increases or decreases with signaling.

Total signaling cost is now:

$$\int_{x*}^{x_{max}} C(x)\mathrm{f}(x)\mathrm{d}x = c\int_{x*}^{1}(1 - x)dx = \frac{c}{2}(1 - x^*)^2. \tag{8}$$

Proposition Two. When all would be in the primary sector if signaling were not feasible, signaling may increase and never maximizes welfare. Signaling is more likely to increase welfare the larger is ω, and, possibly, the smaller is v.

Proof. The rest of this sub-section will prove *Proposition Two*. Now $v\mathrm{E}(x) = v/2$. Using *eqs*.(5) and (8), the change in welfare due to signaling, Ω, is:

$$\Omega = x^*\left(\omega - \frac{vx*}{2}\right) - \frac{c}{2}(1-x^*)^2. \tag{9}$$

Call the level of x^* that would maximize the welfare change with signaling $\tilde{x}^*$. Note, this could result in $\Omega < 0$: no level of signaling improves welfare. I will demonstrate with numerical examples that Ω is positive in some cases. Maximizing Ω:

$$\frac{\partial\Omega}{\partial x*} = \omega - vx^* + c(1-x^*) = 0, \tag{10}$$

$$\frac{\partial^2\Omega}{\partial x*^2} = -(v+c) < 0. \tag{11}$$

Solving *eq*.(10),

$$\tilde{x}^* = \frac{\omega+c}{v+c}. \tag{12}$$

Note that the welfare-maximizing x^* implies some go to the secondary sector with signaling who should and would be in the primary sector with no signaling. This happens if $\tilde{x}^* = \frac{\omega+c}{v+c} > \frac{\omega}{v}$, which is true. Because it is costly, if signaling occurs, fewer individuals should be employed in the primary sector than would be the case with costless information.

Using *eqs*.(4) and (12), $x^* < \tilde{x}^*$ if $v > \omega$, which is true. As was the case when all would be in the secondary sector absent signaling, too many individuals signal, those with $x \in [x^*, \tilde{x}^*)$. Signaling may increase welfare, as shown below, but, in some cases, signaling unambiguously decreases welfare. Interestingly, this occurs when $x^* > \frac{2\omega}{v}$---when *few enough signal* that the output loss (*eq*.(6)) from those with $x \in \left[\frac{\omega}{v}, x*\right]$ who do not signal and are more productive in the primary sector exceeds the output gain (*eq*.(7)) from those with $x \in [0, \frac{\omega}{v})$ who do not signal and are more productive in the secondary sector. Thus, although signaling may not increase welfare, it definitely does not do so if x^* is too large, that is, if too few signal, because then too many are in the secondary sector who have higher output in the primary sector.[18]

I now consider the impact of ω and v on welfare. As demonstrated in the previous section, signaling always increases welfare when all would be in the secondary sector absent signaling, which is true when $\omega > \frac{v}{2}$. This is one reason why a larger ω and a smaller v increase the likelihood that signaling increases welfare.

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[18] As a side note, since $x^* < \tilde{x}^*$, and $\frac{\omega}{v} < \tilde{x}^*$, if x^* happens to equal $\frac{\omega}{v}$, we end up with individuals allocated to sectors as they would be with costless information. This occurs when $c = \frac{v}{2}$. Since $\frac{\partial x*}{\partial c} = \{+\}(v - \omega) > 0$, if $c > \frac{v}{2}$, then $x^* > \frac{\omega}{v}$, so some are in the secondary sector who are more productive in the primary sector. If $c < \frac{v}{2}$, then $x^* < \frac{\omega}{v}$: some are in the primary sector who are more productive in the secondary sector.

Focusing on ω for now, when all would be in the primary sector absent signaling, ω affects Ω in four ways, three of them via x^* (*eqs*.(4), (5), and (9)). With $\frac{\partial x*}{\partial \omega} > 0$, an increase in ω increases the number of individuals who do not signal and are employed in the secondary sector. This raises the output gain from signaling, assuming that $x^* < \frac{2\omega}{v}$, that is, assuming output actually increases with signaling. Also, the cost of signaling falls as fewer signal. Finally, the average output gain from signaling, $\omega - \frac{vx*}{2}$, is reduced as x^* increases, but is directly increased as ω increases. If the net effect on the average output gain from signaling of ω is positive, then an increase in ω unambiguously increases Ω.

$$\frac{\partial\left(\omega - \frac{vx*}{2}\right)}{\partial \omega} = 1 - \frac{v}{v+2c} > 0. \tag{13}$$

Since Ω may be negative, all I can say is that a larger ω makes it more likely signaling increases welfare because a) a larger ω increases welfare with signaling when all would be in the primary sector absent signaling, and b) a larger ω makes it more likely all would be in the secondary sector without signaling, the case when signaling always increases welfare (Section Four).

Now consider the effect of v when all would be in the primary sector with no signaling. Clearly $\frac{\partial x*}{\partial v} < 0$ (using *eq*.(4)). Thus, a lower v means that fewer signal. As discussed previously when ω increases, a lower v also means signaling cost is lower, and there are more in the secondary sector for whom output is higher than in the primary sector (again assuming that

$x^* < \frac{2\omega}{v}$). However, *v* has an ambiguous effect on the average output in the primary sector of those who are in the secondary sector with signaling, $\frac{vx*}{2}$. If this term decreases as *v* decreases, then a decrease in *v* clearly increases *Ω*. I then have:

$$\frac{\partial(vx*)}{\partial v} = \frac{2c}{\{+\}}[2(\omega + c) - 1]. \tag{14}$$

Note that $\frac{\partial x*}{\partial \omega}$ and $\frac{\partial x*}{\partial c}$ are both positive. This is why, the larger either *ω* or *c* is, the more likely it is that $\frac{\partial(vx*)}{\partial v}$ is positive, and this occurs when $\omega + c > ½$. In this case, a lower *v* unambiguously increases *Ω*, which, again, does not mean that $\Omega > 0$.

Table One shows some examples that illustrate the effect of *v*, *ω*, and *c* on the change in welfare from signaling, *Ω*. For two reasons, the assumed values of *v*, *ω*, and *c* are constrained so that $\omega < \frac{v}{2} < \omega + c$. First, in order for all to be the primary sector absent signaling, it must be the case that $\omega < \frac{v}{2}$. Second, if $x^* = 0$, all would signal and remain in the primary sector---where they would be with no signaling. Welfare cannot increase in that case. Therefore, in Table One, I use values of *v*, *ω*, and *c* such that $x^* > 0$, which, from *eq*.(4), requires that $\frac{v}{2} < \omega + c$.

The only cases in which signaling increases welfare---that is, when $\Omega > 0$---are roughly when $\frac{\omega}{v} > .4$. These are examples 12, 17, and 22. As discussed above, a higher value of *ω*, and, possibly, a lower value of v result in a greater value of Ω[19] when $x^* < \frac{2\omega}{v}$. In these three examples, ω is large enough relative to v so that signaling improves welfare. Since in only three of twenty-two cases does signaling increase welfare, it is clear from this and the previous section that the sorting effect of signaling on welfare depends critically on where individuals would be employed if signaling were somehow prohibited.

Additional results involve v and c. Although a lower v does not unambiguously raise welfare, in the four cases in Table One when v decreases, given ω and c, I find that Ω increases. These cases are example 3 versus example 7, example 9 versus example 10, example 15 versus example 18, and example 16 versus example 19.

Also, although a larger c directly reduces welfare, since $\frac{\partial x*}{\partial c} > 0$, a larger *c may* increase Ω by reducing the number who signal. There are nine cases in Table One in which c increases, given ω and v. In five cases, Ω increases as c increases: example 15 versus example 7, example 17 versus example 9, example 20 versus example 10, example 16 versus example 8, and example 21 versus example 11. In four cases, Ω decreases as c increases: example 4 versus example 1, example 22 versus example 12, example 5 versus example 2, and example 6 versus example 3. Thus, there is no consistent relation between signaling cost and the change in welfare with signaling.

6. Technical change and wages

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[19] There is also one case, example 4, when output is actually lower due to signaling.

A higher v would result from skill-biased technical change, something that has been believed to have occurred in recent decades, and has been offered as an explanation for a rising skill gap in earnings (Murphy and Topel, 2016). Supposedly such technical change results in both an increase in wages in skilled jobs, the primary sector in my model, and lower wages in unskilled jobs, my secondary sector. In my model, the wage in the secondary sector equals ω, and does not depend on technical change. I only allow technical change to affect the wage in the primary sector.

The wage in the primary sector equals $\frac{v(1+x*)}{2}$. Now a higher v directly raises the wage in the primary sector, but indirectly lowers the wage by reducing x^*. Using *eq*.(4), I find:

$$\frac{\partial(wage_{\,primary})}{\partial v} = \frac{2c(2c+\omega)}{(v+2c)^2} > 0. \tag{15}$$

Thus, in equilibrium, there are higher earnings in the primary sector as ability in that sector becomes more valuable.

One question (Murphy and Topel, 2016) is why a rising skill gap does not induce even more individuals to pursue higher education. In my model, the higher wage gap equilibrates the market as more signal---those with relatively low ability. Even lower ability individuals have education cost that is too high for them to choose the educational signal.

So far, I have assumed there is no direct effect of education on individual productivity/human capital. I made that assumption because of the skepticism (*c.f.* Caplan, 2011, and forthcoming) regarding education's effect on productivity. Also, I wanted to see if a signaling model can explain behavior including 1) the growing earnings gap between skilled and unskilled jobs, and 2) the increased enrollment in higher education. The model explains both phenomena.

7. Education increases human capital

I now consider the possibility that education increases human capital. Again I assume that productivity is unaffected by individual ability---innate or augmented by human capital---in the secondary sector. First, suppose education increases human capital/productivity in the primary sector by z: productivity = $v(x + z)$. As before, I assume that x is distributed uniformally on the interval [0,1], and that $\mathcal{C}(x) = c(1-x)$. I also assume that $v(\mathrm{E}(x) = \frac{v}{2} < \omega$, so all would be in the secondary sector if education were prohibited. As demonstrated above, this is the case when education increases welfare even if $z = 0$.

As z increases, more individuals will find it optimal to invest in education (x^* decreases), and the socially optimal number who invest in education also increases ($\tilde{x}^*$ decreases). Surprisingly, total welfare does not unambiguously increase as z increases. The ambiguity is because it may be the case that more individuals over-invest in education as the human capital increase from education increases. However, with a uniform distribution of individual ability, total welfare is positively related to z.[20] I now show that more over-invest in education as z increases when ability is distributed uniformally, that is, $\tilde{x}^*- x^*$ is positively related to z.

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[20] The condition for total welfare to increase as z increases is derived in the Appendix.

Now x^* is derived from $v\left(z+\frac{x+1}{2}\right) - c(1\text{-}x) = \omega$, so

$$x^* = max\left(0, \frac{2(\omega+c)-v(1+2z)}{v+2c}\right) \tag{16}$$

The welfare-maximizing level of x^* is found when $x(v + c) - c(1 - x) = \omega$, or

$$\tilde{x}^* = max\left(0, \frac{\omega+c-vz}{v+c}\right), \tag{17}$$

where both $\tilde{x}^*$ and x^* equal the values determined above (*eqs*. (4) and (12)) when $z = 0$. It is easy to show that $\tilde{x}^* > x^*$.[21]

Note, from the numerator of *eq*.(16), if $z > \frac{2(\omega+c)-v}{2v} \equiv \hat{z}$, then $x^* = 0$----all invest in education and are hired in the primary sector. For now assume $z < \hat{z}$. I then have:

$$\tilde{x}^* - x^* = \frac{1}{(v+c)(v+2c)}[(v+2c)(\omega+c-vz) - 2(v+c)(\omega+c) + v(v+c)(1+2z)],$$

$$\text{“ ”} \quad = \{+\}v(v - \omega + vz). \tag{18}$$

Clearly $\tilde{x}^* - x^*$ is positively related to z so, as more human capital is produced with education, more individuals invest in education who should not do so from a social standpoint, even though total welfare increases.

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[21] I find that $\tilde{x}^* > x^*$ if $z > \frac{\omega-c-v}{v}$. With $v > \omega$, $\tilde{x}^* > x^*$.

Alternatively, suppose human capital increases more for those with greater initial ability, x. Then productivity in the primary sector is $v(1+y)x$, with y the increase in productivity/human capital due to obtaining the discrete education level. It is easy to show that, in this case:

$$\tilde{x}^* - x^* = \{+\}v(1+y)(v-\omega) > 0. \tag{19}$$

Clearly an increase in y results in an increase in $\tilde{x}^* - x^*$. Whether education increases human capital by the same amount, or by the same percentage per person, a larger increase in human capital due to education implies there are more who invest in education when it is not socially optimal to do so.

I return to the case when all are more productive by the same amount, z, in the primary sector due to education. Now suppose that $z > \hat{z}$ so $x^* = 0$. Since $\tilde{x}^*$ is inversely related to z, if $z > \hat{z}$, then the number who overinvest in education decreases as z increases. Everyone invests in education in this case, and more should invest in education as z increases.[22] However, as long as all do not invest in education ($z < \hat{z}$), then a greater increase in human capital when education occurs ($dz > 0$) implies an increase in the number who overinvest in education.[23]

8. Summary

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[22] A similar result occurs when education increase human capital by the same percentage for all.

[23] Note, from *eq*.(17), if $z > \frac{\omega+c}{v}$, $\tilde{x}^* = 0$: all should and will invest in education.

I consider a model in which education is not (usually) productive. In contrast to the usual signaling models (*c.f.* Spence, 1974, and Riley, 2001), I assume only one level of education is attainable with a continuum of ability. I find that signaling always occurs, never maximizes welfare, and may increase welfare. Because educational signaling is optimal for a range of ability, excessive signaling occurs by less able individuals, which is consistent with the evidence of increased enrollment in college in the U.S., but which is not the result in most signaling models.

The welfare effects of signaling depend on where individuals would be employed if signaling were prohibited or taxed out of existence. If that is where the more able are more productive, it is likely that signaling does *not* increase welfare. However, it is possible that prohibiting signaling would result in all employed in the sector where the less able are more productive. Then I find that signaling always increases welfare.

Without education as a signal, individuals and firms might be forced to even costlier methods of determining one's ability. Recall (Section One) Gary Becker (1993) suggested that there must be cheaper screening methods than education. Becker was critical of the idea that education was mostly a screening device, and did not enhance one's productivity. Particularly if education plays the dual role of increasing productivity and sorting individuals between jobs (Section Seven), there may be no cheaper alternative to education as a signal.[24]

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[24] Three recent papers offer differing views of the effect of education on productivity. Eble and Hu (2015) analyze educational reform in China that extended the length of primary school by one year. They find large signaling effects of education. More relevant for my model, which implicitly assumes further education implies a baccalaureate degree, Arteaga (2016) considers the leading Colombian university, which reduced the amount of coursework required for a degree in either in either economics or business. She found that human capital accounts for essentially the entire return to education. In contrast, as noted above, Caplan (forthcoming) claims that 80% of the return to higher education in the U.S. is due to signaling. Finally, Bostwick (2016) considers whether field of study in college serves as a signal of ability. She finds that access to elite schools affects individuals' choice of major at non-elite schools, which is consistent with signaling.

I find that a greater increase in human capital when education occurs does not necessarily increase welfare, but does so at least in the case of a uniform distribution of ability. The reason for the general ambiguity in the welfare effect is that there then may be an increase in the number who over-invest in education the more human capital is increased with education.

My main result is that the welfare effects of educational signaling do not simply depend on whether education augments human capital. Of particular importance is what the allocation of individuals to jobs *absent signaling* would be.

Table One. The change in welfare from signaling, Ω, when the alternative is all would be in the primary sector. ($\Omega =$ output change minus cost.)

Example #	v	ω	c	x^*	Output change	Cost	Ω
1	2	.25	1	.125	.016	.383	-.367
2	2	.5	1	.25	.063	.281	-.218
3	2	.75	1	.375	.141	.195	-.054
4	2	.25	1.5	.3	-.015	.368	-.383
5	2	.5	1.5	.4	.04	.27	-.23
6	2	.75	1.5	.5	.125	.188	-.063
7	3	.75	1	.1	.06	.405	-.345
8	3	1	1	.2	.14	.32	-.18
9	3	1.25	1	.3	.24	.245	-.005
10	4	1.25	1	.083	.09	.42	-.33
11	4	1.5	1	.167	.195	.347	-.152

Table One Continued.

Example #	v	ω	c	x^*	Output change	Cost	Ω
12	4	1.75	1	.25	.313	.281	**.32**
13	3	.25	1.5	.083	.01	.63	-.62
14	3	.5	1.5	.167	.063	.521	-.458
15	3	.75	1.5	.25	.094	.422	-.328
16	3	1	1.5	.333	.25	.333	-.083
17	3	1.25	1.5	.417	.26	.255	**.005**
18	4	.75	1.5	.083	.049	.63	-.581
19	4	1	1.5	.143	.102	.551	-.449
20	4	1.25	1.5	.214	.176	.463	-.287
21	4	1.5	1.5	.286	.265	.382	-.117
22	4	1.75	1.5	.357	.37	.31	**.06**

Appendix

Proof that a greater productivity effect of education increases welfare.

This is the case when all would be in the secondary sector if education were prohibited, so education only as a signal improves welfare. Intuitively, if education also is productive, welfare should be even greater. However, more over-invest in education when education is productive, so it remains to be shown that welfare does increase as education is more productive.

Let total welfare with signaling (and not the welfare difference between signaling and no signaling) be denoted by W. Using *eq*.(2), and adding the productivity effect of education, z:

$$W = v\int_{x*}^{x_{max}}(x+z)\,f(x)dx + \omega F(x^*) - \int_{x*}^{x_{max}}\mathcal{C}(x)f(x)dx. \qquad \text{(A1)}$$

Eq.(A1) can be rewritten as:

$$W = [1 - F(x^*)][\,v\mathrm{E}(x+z|x \geq x^*) - \mathrm{E}(\mathcal{C}|x \geq x^*)] + \omega F(x^*). \qquad \text{(A2)}$$

However, x^* is determined by:

$$v\mathrm{E}(x+z|x \geq x^*) - \mathcal{C}(x^*) = \omega. \qquad \text{(A3)}$$

Using *eq*.(A3), *eq*.(A2) becomes:

$$W = \omega + [1 - F(x^*)][\mathcal{C}(x^*) - \mathrm{E}(\mathcal{C}|x \geq x^*)]. \qquad \text{(A2')}$$

Differentiating W w.r.t. z:

$$\frac{\partial W}{\partial z} = \{[\,\mathrm{E}(\mathcal{C}|x \geq x^*) - \mathcal{C}(x^*)]\mathrm{f}(x^*) + [1 - \mathrm{F}(x^*)][\frac{\partial(C(x*))}{\partial x*} - \frac{\partial(E(C|x\geq x*))}{\partial x*}]\}\frac{\partial x*}{\partial z}. \qquad \text{(A4)}$$

With a greater return to investing in education as z increases, clearly $\frac{\partial x*}{\partial z} < 0$. Also, those with more ability, x, have lower education cost by assumption, so $\mathcal{C}(x^*) > \mathrm{E}(\mathcal{C}|x \geq x^*)$, $\frac{\partial(E(C|x\geq x*))}{\partial x*} < 0$, and $\frac{\partial(C(x*))}{\partial x*} < 0$.

Thus the $\{\bullet\}$ term in *eq*.(A4) is unambiguously negative and $\frac{\partial W}{\partial z} > 0$ if $\left|\frac{\partial(E(C|x\geq x*))}{\partial x*}\right| < \left|\frac{\partial(C(x*))}{\partial x*}\right|$.

With the uniform distribution of x in the text, $\frac{\partial(C(x*))}{\partial x*} = -c$, and $\frac{\partial(E(C|x\geq x*))}{\partial x*} = -\frac{c}{2}$, so $\frac{\partial W}{\partial z} > 0$.

Proof that multiple discrete signals will not be observed.

Suppose, as in the text, $x \sim U[0,1]$ and cost = $c(1\text{-}x)$ for one discrete signal. Suppose another signal exists with a cost of $c(1\text{-}x) + F$, where F is a positive constant.
Using the lower cost signal, signaling would occur for those with $x \geq x^*$, where *eq*.(A5) is the same as *eq*.(4) in the text:

$$x^* = \frac{2(\omega+c)-v}{v+2c}\,. \tag{A5}$$

Instead, if more able individuals choose the more costly signal, then those with $x \geq x^{**}$ would signal. Using *eq*.(1), with x distributed uniformally, and with cost now equal to $c(1\text{-}x) + F$:

$$x^{**} = \frac{2(\omega+c+F)-v}{v+2c}\,. \tag{A6}$$

With $x^{**} > x^*$, by choosing the more expensive education, earnings of those with this education rise. Earnings then would equal $v\frac{(1+x**)}{2}$ versus earnings of $v\frac{(1+x*)}{2}$ with the lower cost education. Using *eqs.*(A5) and (A6), the increase in earnings from choosing the higher cost education is $\left(\frac{v}{2}\right)\left(\frac{2F}{v+2c}\right) = \frac{vF}{v+2c} < F$ = the increase in cost from choosing the higher cost signal. Thus, at least with a uniform distribution, and when the additional cost of higher cost signals is a constant, only the lowest cost signal will be chosen by relatively more able individuals. Since other signals have higher cost, and individuals who might choose one of these signals have lower productivity ($x < x^*$), none of them will signal.

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