Appalachian State University

Department of Economics Working Paper

Number 11-16 | September 2011

Spence Revisited: Signaling and the Allocation of Individuals to Jobs

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Signaling and the Allocation of Individuals to Jobs

by

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First Version July 16, 2011

This Version September 2, 2011

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JEL category: D82

Key words: signaling, pooling, Riley outcome, intuitive criterion, and undefeated equilibrium

Abstract

Spence (1974a) considered a variant of his signaling model in which there are two types of jobs, and in which signaling can increase wealth by improving the allocation of individuals to jobs. Using results in signaling games since Spence's work---the *Riley outcome* (Riley, 1979), the *intuitive criterion* (Cho and Kreps, 1987), and *undefeated equilibrium* (Mailath *et al*., 1993)---it is possible to be more precise than Spence was in determining when signaling would occur and what the effect of signaling on wealth would be. We find the likelihood of efficient signaling, inefficient signaling, and pooling equilibria depends on the fraction of more able individuals in the population. With non-trivial gains from job allocation, inefficient signaling does not appear to be the most likely outcome.

1. Introduction

Scholars and pundits continue to debate whether education increases productivity (Leonhardt, 2011). In a study for the Social Science Research Council, Arum *et al*. (2011) find 36% of college students have learned very little after four years. However, even if education does not directly affect productivity, if it is used as a signal of inherent ability, it may increase wealth.

In his classic work on market signaling, Spence (1974a) considered a world in which more able individuals would signal their ability in order to be paid more. If there is only one type of job, and signaling, say via education, does not directly affect individual productivity, there is no output gain from signaling---wealth is simply redistributed from the less able to the more able.[1]

Spence (1974a) also considered a variant of his basic signaling model in which there are two types of jobs, and in which signaling can increase wealth by improving the allocation of individuals to jobs. As discussed below, when the allocation of individuals to jobs matters, Spence found signaling may increase or decrease wealth.

Since Spence's initial analysis of signaling, there have been several results in signaling models. These include the idea of the lowest cost signaling equilibrium, the *Riley outcome* (Riley, 1979), the *intuitive criterion* (Cho and Kreps, 1987), and *undefeated equilibrium* (Mailath *et al*., 1993). All of these results will be discussed below. When using these results, it is possible

[1] Spence (2002) considered the case when education is a signal and it increases individual productivity. Then all individuals would invest in some education, even with perfect information. If the marginal cost of signaling is significantly larger for less able individuals than for the more able, excessive investment in education would not occur; the less able would not desire to mimic the education level of the more able, even if doing so resulted in the former being paid as if they were more able. With a small enough difference in the marginal cost of signaling, the more able would have to invest in a larger amount of education than they would with perfect information. Leppämäki and Mustonen (2009) consider a model where the signal may create either a positive or negative product market externality.

to be more precise than Spence was in determining when signaling would occur and what the effect of signaling on wealth would be. These results do not appear to have been used previously[2] to consider the Spence model when job allocation matters.[3] Most signaling research has focused on the existence of equilibrium.[4] The goal of this paper is to apply the *Riley outcome*, the *intuitive criterion*, and *undefeated equilibrium* to essentially the same model used by Spence (1974a).[5]

Using the *Riley outcome*, the *intuitive criterion*, and *undefeated equilibrium*, one can address two important issues in Spence's analysis of job assignment. First, Spence allows for multiple signaling equilibria.[6] He mentions the possibility of firms competing and lowering the required education level as much as possible, but suggests there is not much incentive for them to do so (Spence, 1974a, p.170). Cho and Kreps (1987) argue only a separating equilibrium with the lowest possible level of the signal, the *Riley outcome*, survives their *intuitive criterion*.[7]

Second, Spence was not clear on when pooling equilibria might occur, arguing we might have such equilibria "...either by fiat, or naturally as an equilibrium..." (Spence, 1974a, p.156). With the concept of *undefeated equilibrium*, Mailath *et al.* (1993) refine the *intuitive criterion*, which rules out all pooling equilibria, and demonstrate pooling should only occur when the more

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[2] Greenberg (1989) modeled job allocation, and found signaling might result in inefficient allocation of individuals to jobs. His results are based on the possibility of multiple signaling equilibria (ruled out by the *intuitive criterion*), and on not considering when individuals would deviate from a pooling equilibrium (*undefeated equilibrium*).

[3] A different job allocation literature was begun by Waldman (1984). In his model, job assignment within a firm is used by potential employers as a signal of an individual's ability. For a model that combines education as a test and job assignment as a a signal, see Perri (1993).

[4] Among the large literature on existence of equilibrium in signaling games are Riley (1975, 1979, and 2002), Cho and Kreps (1987), Mailath *et al*. (1993), and Cai *et al*. (2007).

[5] Spence (1974b) considered a model of job allocation in which individuals are continuously distributed with respect to ability. The results are similar to those with two types of ability, so the latter is the focus herein.

[6] In a later paper (1976), Spence considered competition in the credentials dimension which eliminated all but the Pareto dominant signaling equilibrium, but did so only with certain production functions. Thus, multiple signaling equilibria were not ruled out in general.

[7] Riley (1975) argued 1) only the signaling equilibrium with the lowest level of signaling would survive experimentation by buyers, and 2) even that equilibrium might not survive. Riley (1979) suggests different assumptions can result in survival of the signaling equilibrium with the lowest level of signaling, as does the work of Cho and Kreps (1987).

able are better off pooling (with no investment in the signal) than they would be in a separating equilibrium with the with the lowest level of signaling.[8] As will be seen, we should expect pooling to occur when the fraction of more able individuals in the population, α, is sufficiently large.

One recent paper that did consider a signaling model with more than one job is Hopkins (forthcoming). He analyzed a matching model in which the signal at least identifies individuals inherent ability, and may increase the value of the match. In this model, individuals do not consider the benefit of their signal to potential partners, and, for this reason, may underinvest in the signal. Thus, signaling in a matching environment may involve externalities that have opposite effects. In a job allocation model, there are no offsetting effects. What distinguishes the latter type of model is the social and private returns to signaling are not the same, so we may have efficient or inefficient signaling.

We consider Spence's model of job allocation in the next section. Using Spence's four basic assumptions, along with the *Riley outcome*, the *intuitive criterion*, and *undefeated equilibrium*, we show the following in Section 3: when α is relatively small, we get an equilibrium with signaling that improves wealth; when α is somewhat larger, we get a signaling equilibrium that lowers wealth; and for the largest values of α, we get a pooling equilibrium when pooling involves higher wealth than we would have with signaling. The likelihood of these different scenarios is considered in Section 4, and a summary is contained in Section 5.

2. The Spence Model of Job Allocation

[8] See the Appendix for a brief discussion of the *intuitive criterion* and *undefeated equilibrium*.

In his dissertation, Spence (1974a) apparently did not view the job allocation problem as having great importance, since he relegated the analysis of the problem to an appendix.[9] Spence apparently did not think the problem was of too much interest because the results were ambiguous. We find signaling may or may not improve wealth, but we also show the results are based on the share of more able individuals in the population, α.

The job allocation model we use is essentially that found in Spence (1974a). We assume two types of individuals, *more able* and *less able,* and two types of jobs, *skilled* and *unskilled*.[10] The (constant) marginal revenue products (MRPs) of individuals in different jobs are listed in Table One.

Table One. MRPs.		
	More Able	Less Able
Skilled Jobs	$e\theta$	θ
Unskilled Jobs	$a\theta$	$a\theta$

Spence used four basic assumptions, which we also employ. The first three assumptions refer to Table One, where $\theta > 0$.

Assumption One. The more able are more productive in the skilled job than are the less able. This requires $e > 1$.

Assumption Two. The more able are more productive in skilled jobs than are the less able in unskilled jobs. This requires $e > a$.

Assumption Three. The less able are more productive in unskilled jobs than they are in skilled jobs. This requires $a > 1$.

[9] In his book, Spence says he considered the problem of job allocation "...largely in response to suggestions and questions raised by Zvi Griliches and George Stigler..." (1974a). He also devoted only four of thirty-seven pages in a journal article on signaling to job allocation (Spence, 1974b).
[10] In Spence (1974a), more able individuals are called Group 2, less able individuals are called Group 1, the skilled job is called Job 2, and the unskilled job is called Job 1.

Assumption Four. *The marginal cost of signaling is lower for the more able than it is for the less able*. With the level of the signal denoted by y, assume the cost of signaling is y for the less able, and y/g for the more able, with $g > 1$.[11]

An additional question is the productivity of the two types of individuals in the unskilled job. Spence considered two possibilities: 1) the more able have a larger MRP in the unskilled job than the less able, and 2) the opposite of 1). From Table One, we assume both types have the same productivity in the unskilled job. There are three reasons for this assumption. First, it is essentially the average of what Spence considered. Second, it allows us to reduce possible cases to consider. Third, and most importantly, ability should be of less importance in an unskilled job. We simply assume it is of no ability. In the next section, we briefly consider what happens if more and less able individuals do not have the same productivity in the unskilled job.

Before proceeding to our model of job allocation, for purposes of comparison, consider the results in Spence (1974a). First, consider the case when pooling (with $y = 0$) would result in all placed in the unskilled job. Using our notation, Spence found signaling increases wealth (is efficient) if e-a and g are high enough, and the signal level is low enough. In contrast, for this case, we find wealth *always* is increased with signaling. Second, when pooling would result in all placed in the unskilled job, Spence found, when signaling benefits the less able, it also benefits the more able, so wealth clearly increases. In this case, we find the following.

- Signaling can not benefit the less able, since they will be paid less than the pooling wage if the more able signal.
- Signaling may benefit the more able. When it does, it may or may not increase wealth, depending on α.

[11] One criticism of signaling models is the assumption signaling cost and productivity are inversely related. Such an assumption is not necessary if individuals receive a "grade" in addition to the level of the signal, and the more able are more likely to get a good grade (Weiss, 1983). For simplicity, and to follow the original model in Spence (1974a), we ignore grades and maintain the assumption the more able have a lower marginal cost of signaling.

• Pooling may occur. If pooling does occur, it yields greater wealth than would signaling.

3. A job allocation model

A. Outline of the game

Most details of the model are found in Assumptions One through Four and in Table One in Section 2. Further, suppose there are N individuals in the population. Individuals and employers know α of these individuals are more able. If signaling does not occur, all are paid the same wage equal to W_{pool}, with $W_{pool} = max\{(\alpha e + 1\text{-}\alpha)\theta, a\theta\}$. The Riley outcome is when less able individuals set $y = 0$, and more able individuals set $y = y_{Riley}$---the lowest level of the signal that induces a separating equilibrium. Individuals move first. If more able individuals do not signal, firms respond and hire individuals in the job where expected productivity is the highest, with pay of $max\{(\alpha e + 1\text{-}\alpha)\theta, a\theta\}$. If signaling occurs, more able individuals set $y = y_{Riley}$. Less able individuals set $y = 0$. In the case of signaling, firms respond by hiring those with $y = y_{Riley}$ in skilled jobs at a wage of $g\theta$, and those with $y = 0$ in unskilled jobs at a wage of $a\theta$.

Using the *intuitive criterion* (Cho and Kreps, 1987), we assume signaling only occurs at the *Riley outcome*. Further, using the notion of *undefeated equilibrium* (Mailath *et al*., 1993), signaling only occurs if it is preferred by the more able individuals to pooling at $y = 0$.[12]

B. Pooling in unskilled jobs

If $a\theta > (\alpha e + 1\text{-}\alpha)\theta$, expected productivity with no signaling is greater in unskilled jobs than in skilled jobs. This occurs when $\alpha < \alpha^*$:

[12]Cai *et al*. (2007) consider an equilibrium refinement they call the *Local Credibility Test* in a signaling game. They consider amultiple types and imperfect negative correlation between productivity and the marginal cost of signaling. As with *undefeated equilibrium*, the *Local Credibility Test* is more likely to select the *Riley outcome* the lower the percentage of more able individuals in the population.

$$\alpha^* = \frac{a-1}{e-1}. \tag{1}$$

With $a < e$ and both a and $e > 1$, clearly $0 < \alpha^* < 1$. Now we show signaling will always occur in this case. Ignore pooling for now. In a signaling equilibrium, those who signal will go to skilled jobs and be paid $e\theta$, and those who do not signal will go to unskilled jobs and be paid $a\theta$. For signaling to occur, more able individuals must prefer to be viewed correctly, and not as less able, and the less able must not want to mimic the more able. We must have:

$$e\theta - y/g \geq a\theta, \tag{2}$$

$$e\theta - y < a\theta. \tag{3}$$

Ineqs.(2) and (3) yield:

$$(e\text{-}a)\theta < y \leq (e\text{-}a)\theta g. \tag{4}$$

With $y = y_{Riley}$, the lowest level of the signal that induces a separating/signaling equilibrium:

$$y_{Riley} \approx (e\text{-}a)\theta. \tag{5}$$

The payoff to more able individuals from signaling is

$(e\text{-}a)\theta$ - y_{Riley}/g = $(e\text{-}a)\theta$ - $\frac{(e-a)\theta}{g}$ = $(e\text{-}a)\theta\left(1-\frac{1}{g}\right)$ > 0. However, this is also the condition for signaling to increase wealth. Because of the assumption all are worth $a\theta$ in unskilled jobs, the pooling wage in unskilled jobs is the same as the wage one gets in a signaling equilibrium if one does not signal. Signaling occurs and increases wealth because the signal results in the reallocation of more able individuals from unskilled jobs to skilled jobs were they are more productive, and the output gain exceeds the cost. The private return to signaling, $(e\text{-}a)\theta$, also equals the social return.

As discussed in Section 2, Spence considered the possibility individuals did not have the same productivity in unskilled jobs. We briefly consider that case now.[13] With the productivity of less able individuals in unskilled jobs equal to $a\theta$, there are two possibilities.

First, suppose more able individuals have productivity of $b\theta$ in unskilled jobs, with $b > a$. Let the expected MRP in unskilled jobs equal $\overline{MRP}_{unskilled}$. Now $b\theta > \overline{MRP}_{unskilled}$. The social benefit to signaling when pooling is in unskilled jobs is the output gain from more able individuals moving from unskilled jobs to skilled jobs, $e\theta$ - $b\theta$. However, $e\theta$ - $b\theta$ < $e\theta$ - $\overline{MRP}_{unskilled}$ = the private benefit from signaling, so too many will signal: for some values of α, signaling occurs when it reduces wealth.

Second, if productivity of more able individuals in unskilled jobs is $d\theta$, $d < a$, then $d\theta < \overline{MRP}_{unskilled}$. The social benefit to signaling is $e\theta$ - $d\theta$ > $e\theta$ - $\overline{MRP}_{unskilled}$ = the private benefit, so too few will signal: for some values of α, signaling will not occur when it would increase wealth.

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[13] Another variation, used in Spence (1981), is when the more able have the same productivity in unskilled jobs as the less able have in skilled jobs. Spence did not justify this assumption, but it is simply another way of introducing a social return to allocating individuals to jobs. It seems more plausible different types of individuals would have the same productivity in the unskilled job (as assumed herein) due to a particular skill that is required for skilled jobs (of which the more able have more than the less able), with said skill of no value in unskilled jobs.

Since we will find cases of both inefficient and efficient signaling when all have the same productivity in unskilled jobs, we will focus on that case in the rest of the paper---that is, we assume all have productivity of $a\theta$ in the unskilled sector.

C. When signaling increases wealth

When $\alpha > \alpha^*$, pooling would involve all being employed in skilled jobs. In the next sub-section, we will apply the concept of *undefeated equilibrium* (Mailath *et al*., 1993) to see when more able individuals prefer signaling to pooling. Assuming for now signaling occurs, we consider when it would increase wealth. Now the social return to signaling comes from moving less able individuals from the skilled sector to the unskilled sector, with an output gain of $(a\text{-}1)\theta$ per less able individual. Thus the total gain to society from signaling is $(a\text{-}1)\theta(1\text{-}\alpha)N$. The social cost of signaling is the amount spent by the more able individuals, which equals $\frac{(e-a)\theta\alpha N}{g}$. When pooling is at skilled jobs, signaling increases wealth if $\alpha < \alpha^{**}$:

$$\alpha^{**} = \frac{g(a-1)}{g(a-1)+e-a} \,. \qquad (6)$$

Clearly $\alpha^{**} < 1$. Also, for $\alpha^{**} > \alpha^*$, we must have $e > a$, which is true. We will see below signaling will occur if $\alpha < \alpha^{**}$. Thus, if $\alpha^* < \alpha < \alpha^{**}$, signaling occurs and increases wealth.

The private gain to more able individuals from signaling is the increased pay that results, $[e\theta - (\alpha e + 1\text{-}\alpha)\theta]\alpha N$. As shown above, the social gain from signaling is $(a\text{-}1)\theta(1\text{-}\alpha)N$, resulting from reallocating the less able from skilled jobs to unskilled jobs. The effect of a higher α on the private gain from schooling is ambiguous. As α increases, the pooling wage rises, so there is less of a return per individual, but there are more of the more able individuals. An increase in α lowers the social gain from signaling because there are fewer less able individuals to reallocate to unskilled jobs. For $\alpha > \alpha^{**}$, wealth decreases if signaling occurs.

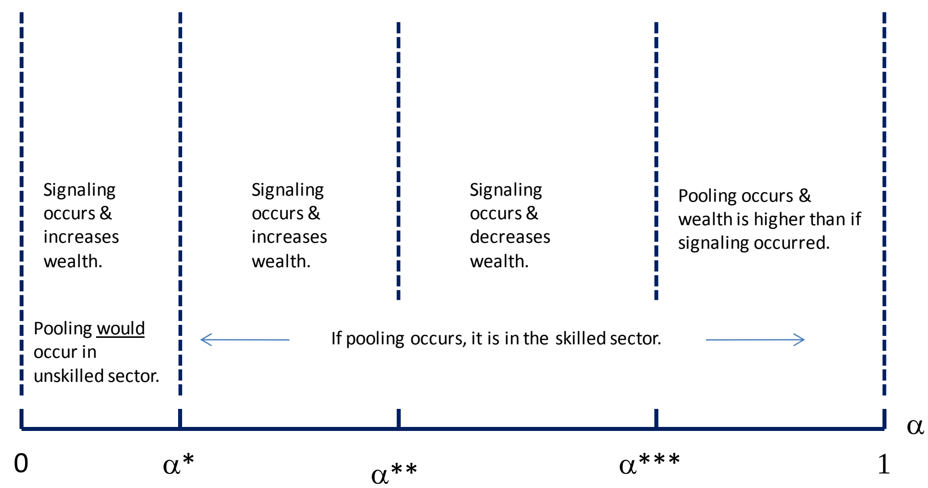
D. When pooling is preferred to signaling

To determine when pooling will occur when it involves all going to skilled jobs, we use *undefeated equilibrium* (Mailath *et al*., 1993) to refine the *intuitive criterion* (Cho and Kreps, 1987). *Undefeated equilibrium* and the *intuitive criterion* imply more able individuals will deviate from pooling when they are better off by signaling with $y = y_{Riley}$. Thus, these individuals will prefer signaling to pooling if $e\theta - \frac{(e-a)\theta}{g} > (e\alpha + 1-\alpha)\theta$. Signaling will occur if $\alpha < \alpha^{***}$:

$$\alpha^{***} = \frac{g(e-1)+a-e}{g(e-1)}. \qquad (7)$$

With $a < e$, clearly $\alpha^{***} < 1$. Also, $\alpha^{**} < \alpha^{***}$ if $0 < (a-e)^2(g-1)$, which is true. The possible results are shown in Figure One. Note the actual width of the ranges shown in Figure One depends on *e*, *a*, and *g*, as will be discussed in the next section.

Figure One



We can use Figure One to summarize the results so far. For low enough values of α, $\alpha < \alpha^*$, signaling occurs and increases wealth because the alternative is pooling in unskilled jobs. The output gain from signaling involves reallocating more able individuals from unskilled jobs to skilled jobs. With the assumption all have the same productivity in unskilled jobs, the private return to signaling---the wage increase for the more able---equals the social return.

For somewhat higher values of α, $\alpha^* < \alpha$, pooling would be in skilled jobs. There is a social gain from signaling---the reallocation of the less able from skilled jobs to unskilled jobs---but this differs from the private gain---increased earnings for the more able.

For $\alpha^* < \alpha < \alpha^{**}$, the social gain from signaling exceeds the (social) cost, but for higher values of α, $\alpha^{**} < \alpha < \alpha^{***}$, when there are relatively few unskilled individuals, the social gain from signaling is less than the cost. For even higher values for α, $\alpha > \alpha^{***}$, the more able will not deviate from pooling, and wealth is higher than if signaling occurred.

Thus, only if $\alpha^{**} < \alpha < \alpha^{***}$ will we have inefficient signaling.

4. The likelihood of efficient signaling, inefficient signaling, and pooling

A. Basics

We found three basic possibilities which depend on α. There is a range for α when signaling efficiently occurs. Call this range r_e, with $r_e = \alpha^{**}$. There is a range in which signaling occurs and is inefficient. This range is r_i, with $r_i = \alpha^{***} - \alpha^{**}$. Finally, there is a range when pooling occurs and wealth is higher than if signaling occurred. This range is r_p, with $r_p = 1 - \alpha^{***}$.

B. A change in the marginal cost of signaling for the more able, g

Using *eqs*. (1), (6), and (7), we have:

$$\frac{\partial r_e}{\partial g} = \{+\}(e-a)(a-1) > 0, \tag{8}$$

$$\frac{\partial r_i}{\partial g} = \{+\}[g(a-1)(2-g) + e - a], \tag{9}$$

$$\frac{\partial r_p}{\partial g} = -\{+\}(e-1)(e-a) < 0. \tag{10}$$

Now $\frac{\partial r_i}{\partial g} > 0$ if $g < 2$. If g increases, the marginal cost of signaling for more able individuals decreases. Since $\frac{\partial \alpha^{***}}{\partial g} > 0$, the pooling range, r_p, falls as g increases and more able individuals find signaling preferable to pooling for larger values of α. Also, the range of efficient signaling increases as signaling is less costly---$\frac{\partial \alpha^{**}}{\partial g} > 0$. Thus, the residual, the range of inefficient signaling, could increase or decrease as g increases. For low enough values of g

($g < 2$), a lower marginal cost of signaling increases the range of α for which inefficient signaling occurs.

C. A change in the productivity of the more able in skilled jobs, e

Again using *eqs*. (1), (6), and (7):

$$\frac{\partial r_e}{\partial e} = -\{+\}g(a\text{-}1) < 0, \tag{11}$$

$$\frac{\partial r_i}{\partial e} = \{+\}(e\text{-}a)(a\text{-}1)[g(e + a - 2) + e - a] > 0, \tag{12}$$

$$\frac{\partial r_p}{\partial e} = \{+\}g(a\text{-}1) > 0. \tag{13}$$

An increase in productivity of the more able in skilled jobs, *e*, reduces the range for efficient signaling for two reasons. First, efficient signaling occurs when pooling would be in unskilled jobs. A larger *e* increases the pooling wage in skilled jobs, so we are less likely to have pooling in unskilled jobs---α* falls. Second, when pooling is in skilled jobs, the productivity of the more able has no impact on the social gain since, with pooling or signaling, the more able would be in skilled jobs. However, an increase in *e* increases y_{Riley}, so signaling cost increases. Thus, the maximum value of α for which signaling is efficient, α**, falls, another reason r_e decreases. Since the pooling wage in skilled jobs rises as *e* increases, the pooling range also increases. With r_e decreasing and r_p increasing, in principle, the range for inefficient signaling could rise or fall, but in fact $\frac{\partial r_i}{\partial e} > 0$.

D. A change in the productivity of the less able in unskilled jobs, a

Using *eqs*. (1), (6), and (7):

$$\frac{\partial r_e}{\partial a} = \{+\}g(e\text{-}a) > 0, \tag{14}$$

$$\frac{\partial r_i}{\partial a} = -\{+\}(e\text{-}a)[2(e\text{-}1)g(a\text{-}1) + 3(e\text{-}1)(e\text{-}a) + g(e\text{-}a)(a\text{-}1) + (e\text{-}a)^2] < 0, \tag{15}$$

$$\frac{\partial r_p}{\partial a} = -\{+\} < 0. \tag{16}$$

If the less able are more productive in unskilled jobs ($da > 0$), the range of efficient signaling increases. The social gain from signaling is higher when the alternative is pooling in skilled jobs since it involves reallocating less able individuals from skilled to unskilled jobs where they are now more productive. Since an increase in a lowers y_{Riley}, more able individuals prefer signaling to pooling for a greater range of α. Thus, $d\alpha^{***} > 0$, and $dr_p < 0$. An increase in r_e and a decrease in r_p could imply an uncertain effect on r_i, but we have $\frac{\partial r_i}{\partial a} < 0$.

Summarizing the results, we find inefficient signaling is more likely the lower the marginal cost of signaling for the more able, the higher the productivity in skilled jobs for the more able, and the lower the productivity of the less able in unskilled jobs. As discussed previously, when pooling would result in all placed in the unskilled job, Spence found signaling increases wealth if e and g are high enough, a is low enough, and the signal level is low enough. We find wealth *always* is increased with signaling in this case.

When pooling would result in all placed in skilled jobs, we get essentially the opposite results Spence found in his model when pooling would result in all placed in unskilled jobs: signaling is less likely to be efficient when e and g are high and a is low. Also, with pooling in skilled jobs, Spence found, when signaling benefits the less able, it also benefits the more able, so wealth clearly increases. We find signaling can not benefit the less able, since they will be paid less than the pooling wage if the more able signal; signaling may benefit the more able, and, when it does, it may or may not increase wealth, depending on α; and pooling may occur. If pooling does occur, it yields greater wealth than would signaling.

Thus, using the *Riley outcome*, the *intuitive criterion*, and *undefeated equilibrium* allows different and more precise results.

E. Numerical examples

We have seen the effect of the variables *e*, *g*, and *a* on the likelihood of equilibria with efficient signaling, inefficient signaling, and pooling. To get an idea of the magnitude of the ranges for these possibilities, consider Table Two.

Table Two. Values for α*, α**, and α***.

e	*a*	*g*	α*	α**	α***	Comment
2	1.01	2	.01	.02	.51	Approximately the basic Spence model with 1 job.
2	1.1	2	.1	.18	.55	
2	1.1	1.5	.1	.14	.4	
2	1.25	2	.25	.4	.625	
2	1.25	1.5	.25	.33	.5	
2	1.5	1.5	.5	.6	.67	
2	1.5	1.25	.5	.56	.6	
2	1.1	1.1	.1	.11	.18	
1.5	1.01	1.5	.02	.03	.35	Approximately the basic Spence model with 1 job.
1.5	1.1	1.5	.2	.27	.47	
1.5	1.25	1.5	.5	.6	.67	

The first and ninth rows in Table Two show results when *a* = 1.01, which means a less able individual is essentially as productive in either type of job. Also, in a signaling equilibrium, an individual's alternative to signaling---going to an unskilled job for a wage of $a\theta$---is almost the same as what it would be if the individual were placed in the skilled job and viewed as less able, θ. This is essentially the basic Spence model (1974a) when there is one type of job. Since *a* is so close to one, for even a very small value for α, $max\{(\alpha e + 1\text{-}\alpha)\theta, a\theta\} = (\alpha e + 1\text{-}\alpha)\theta$--- pooling is in the skilled job. Thus, the unskilled job is almost irrelevant, as evidenced by the values of α* for these two examples, .01 and .02.

The example in the first row is virtually identical to the one-job example prominently featured in Spence (1974a): the more able are twice as productive as the less able and have a marginal cost of signaling that is ½ that of the less able. However, even with almost no possibility for efficient signaling (r_e = .02), the pooling range is the same size as the range for inefficient signaling ($r_p = r_i$ = .49). This, result is due to the *undefeated equilibrium* refinement choosing pooling when the more able would be worse off deviating to a signaling equilibrium with $y = y_{Riley}$.

With a = 1.01, if the productivity advantage in skilled jobs and marginal cost advantage to the more able is lowered somewhat, there still is little possibility of efficient signaling. With $e = g = 1.5$, r_e is only .03. However, as shown above if, e or g is reduced (assuming $g < 2$), r_i falls. We now have r_i = .32 and r_p = .65. Thus, pooling is twice as likely to occur than inefficient signaling, so, even with essentially a world of one job type, inefficient signaling is not necessarily the most likely outcome. Only when e and g are high and a is low (close to one) would we find r_i comparable to the sum of r_e and r_p.

With any non-trivial gain from job assignment, the results change significantly. For example, compare rows one and four in Table Two. The only difference between the two cases in, from row one to row four, a increases from 1.01 to 1.25. In row four, there is a 25% productivity gain from less able individuals moving from skilled jobs to unskilled jobs. Note this is only ¼ the assumed productivity difference between the more and less able in skilled jobs.

Now, $r_e = .4$, $r_p = .375$, and $r_i = .225$. The range for efficient signaling is more than 50% larger than that for inefficient signaling, and the combined range for efficient signaling and (efficient) pooling is more than three times the range for inefficient signaling.

Finally, somewhat lower values for e and g, along with a non-trivial value for a, have a significant effect on the range of inefficient signaling. For example, suppose $e = g = 1.5$, and $a = 1.25$, so the productivity advantage of the less able in unskilled jobs versus skilled jobs is ½ the productivity difference between the more and less able in skilled jobs (last row in Table Four). Now $r_e = .6$, $r_i = .07$, and $r_p = .33$. The efficient signaling range is more than eight times the size of the inefficient signaling range, and the combined range for efficient signaling and pooling is thirteen times the range for inefficient signaling. Thus, it is plausible the likelihood of inefficient signaling is not considerable.

5. Summary

A potentially important social return to educational is in the ability to identify individuals' productivity in order to optimally assign them to jobs. Using results for equilibrium refinements in signaling games developed since Spence (1974a) considered a model of job allocation, it is possible to be more precise than Spence was in determining when signaling would occur and what the effect of signaling on wealth would be. We find inefficient signaling is more likely the lower the marginal cost of signaling for the more able, the higher the productivity in skilled jobs for the more able, and the lower the productivity of the less able in unskilled jobs.

In contrast to Spence's earlier results, we find a clear effect of the fraction of more able individuals in the population on the equilibrium. Small values for this fraction are likely to be associated with efficient signaling; somewhat larger values of the fraction would imply inefficient signaling, and the highest values of the fraction would result in (efficient) pooling equilibrium. The larger the possible gain from job allocation, the less likely inefficient signaling occurs.

One implication of these results is higher education might serve a valuable signaling function even if little human capital were acquired in universities. Except for the original work by Spence (1974a, 1974b), there has been little focus on the social gain from education when the educational signal can improve the allocation of individuals to jobs. For example, in his comparison of human capital and signaling explanations of wages, Weiss (1995) suggests signaling models of education are resisted by some economists because these models imply inefficient equilibria since private returns to education exceed social returns. The results herein suggest claims of little human capital accumulation in higher education, while not to be ignored, should be considered given other potential social benefits of higher education---including its role in improving job allocation.

Appendix

The Intuitive Criterion and Undefeated Equilibria.

For the simplest explanation of the *intuitive criterion* (Cho and Kreps, 1987), consider the case with only one job. More able individuals have productivity equal to 2 and a cost of signaling of $y/2$. Less able individuals have productivity equal to 1 and signaling cost of y. The fraction of more able individuals in the population is α. Thus the pooling wage is $W_{pool} = 2\alpha + 1\text{-}\alpha = 1 + \alpha$. Start with the case when $\alpha = .75$. From Figure A1, if one assumes (as with the *intuitive criterion*) the less able believe pooling remains available to them even if the more able signal to separate themselves, then the shaded area contains separating equilibria preferred by the more able to pooling but not preferred by the less able. However, if the less able realize signaling by the more able implies a wage = 1 for those who do not signal, then the relevant level of utility for the less able is given by $U^{separate}_{less\ able}$. Thus, the less able would prefer to signal and be paid 2 as long as $y < 1 = y_{Riley}$, point I_3. Since point I_3 is below $U^{pool}_{more\ able}$, more able individuals will not deviate from the pooling equilibrium with $y = 0$ and a wage of 1.75. Pooling survives the *undefeated equilibrium* refinement if y_{Riley} is <u>not</u> contained in the area in which the more able will deviate from pooling when the less able would not deviate when they believe pooling remains available to them. In Figure A2, with $\alpha = .5$, y_{Riley} (point J_2) is contained in the shaded area. In essence, a pooling equilibrium does not survive if the more able are better off signaling with $y = y_{Riley}$ than they are with pooling and $y = 0$.

Figure A1. $\alpha = .75$

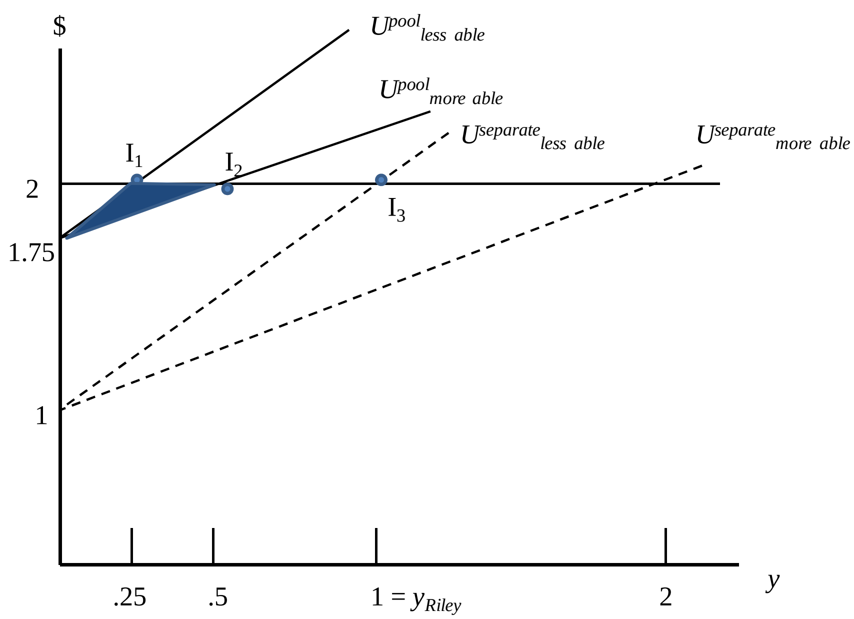
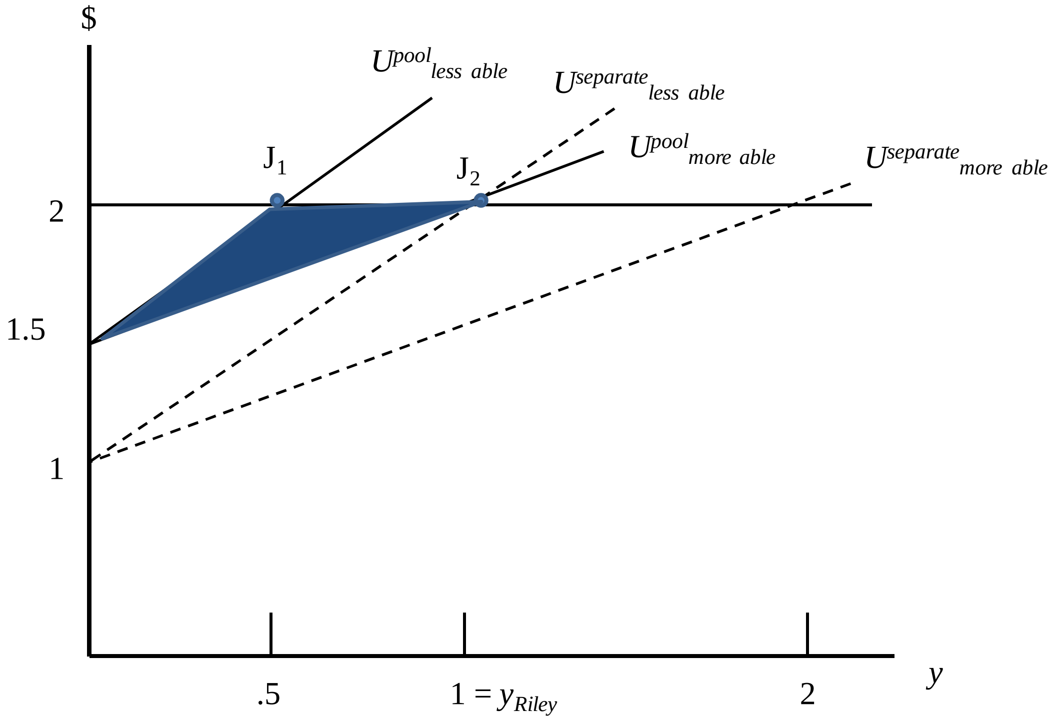


Figure A2. $\alpha = .5$



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