Cross-country Variation in Factor Shares and its Implications for Development Accounting*

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Abstract

The stability of factor shares has long been considered one of the "stylized facts" of macroeconomics. However, the relationship between cross-country factor shares and economic development is dependent on how factor shares are measured. Most factor share studies acknowledge only two factors of production: total capital and total labor. The failure to acknowledge more than two factors yields misleading results. Recent theoretical work predicts a systematic relationship between the stage of economic development and non-reproducible and reproducible factor shares. I disentangle physical capital's share from natural capital's share, and I disentangle human capital's share from unskilled labor's share. The results reveal that non-reproducible factor shares decrease with the stage of economic development, and reproducible factor shares increase with the stage of economic development. Studies relying on the macroeconomic paradigm of constant factor shares should be revisited. Development accounting nearly always assumes the constancy of factor shares. I perform the development accounting exercise but allow factor shares to vary and distinguish between reproducible and non-reproducible factors. My approach yields results that stand in stark contrast to those previously attained. The general consensus is that at least half of the cross-country variation in output per worker is attributable to cross-country variation in the TFP residual. With my approach, the majority of variation in output per worker accrues to factor shares, specifically physical capital's share and natural capital's share. TFP's explanatory power decreases by more than 30 percentage points. This evidence does not, however, diminish the role of technical change. Rather, the evidence indicates the importance of acknowledging a new type of technical change, one that impacts factor shares.

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1 Introduction

Capital shares and labor shares are typically treated as constant parameters. For example, Hall and Jones (1999), in an investigation of the role of productivity in explaining cross-country differences in output per worker, assume that capital shares and labor shares are constant across countries and equal to 1/3 and 2/3 respectively. Some studies, such as Gollin (2002), present empirical evidence in support of constant factor shares across countries. Others, such as Zuleta (2008a), conclude that factor shares vary across countries. Despite conflicting empirical evidence and despite the doubts about the constancy of factor shares expressed by Keynes (1939) and Solow (1958), most researchers accept Kaldor's (1961) submission that factor shares are constant as a "stylized fact" of macroeconomics.

Factor shares are not constant when factors of production are properly defined and measured. The key step is making a distinction between reproducible factors and non-reproducible factors. In most factor share studies, only two factors of production, capital and labor, are acknowledged. Failure to acknowledge more than two factors yields results and conclusions that are misleading at best. When discussing capital, economists generally refer to physical or human capital—physical capital being tools, machinery, and structures, and human capital encompassing education, health, and training. However, standard capital share measures include the fractions of income paid to physical capital as well as natural capital, which encompasses all natural resources including land, minerals, and oil. Physical capital and natural capital are two distinct factors. Physical capital is reproducible, meaning it can be accumulated, whereas natural capital is non-reproducible and can not be accumulated[1]. Therefore, any claim about the standard capital share and how it relates to the stage of economic development is really

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[1] Non-reproducible factors are those factors with which an economy is endowed. Reproducible factors have to be produced.

a claim about two separate factor shares and their collective relationship with the stage of economic development. Likewise, standard measures of labor's share entangle the fraction of income paid to human capital, a reproducible factor, and unskilled labor, a non-reproducible factor.

In the first part of this paper, I disentangle physical capital's share from natural capital's share and human capital's share from unskilled labor's share. There is strong evidence that non-reproducible factor shares decrease with the stage of economic development, and reproducible factor shares increase with the stage of economic development. This finding has theoretical and empirical implications. First, it provides support for theoretical growth models, such as those presented by Peretto and Seater (2008) and Zuleta (2008b), that incorporate factor eliminating technical progress. Secondly, it suggests that any theoretical or empirical study relying on Kaldor's claim that factor shares are constant should be revisited.

One macroeconomic exercise that virtually always assumes constancy of factor shares is the estimation of Total Factor Productivity (TFP). Examples in the literature include Klenow and Rodriguez-Clare (1997), Hall and Jones (1999), and Caselli (2005). The second part of this paper looks at the implications of systematic variation in factor shares for the measurement of TFP across countries. Specifically, I compare the fraction of cross-country variation in economic performance attributable to variation in TFP to the fraction of cross-country variation in economic performance attributable to variation in factors and factor shares. Rather than assume factor shares are constant across countries, I allow factor shares to vary in accordance with the estimates presented in the first part of the paper.

TFP is generally thought to account for at least half of the variation in output per worker. I find that the majority of variation in output per worker accrues to factor shares when factor shares are allowed to vary and a distinction between reproducible and non-reproducible factors is made. Variation in output per worker accruing to TFP falls by about 32 percentage points. These results, in addition to suggesting that factor shares play an important role in development accounting, reveal the inappropriateness of forcing all technical progress to work through the TFP residual. Technical progress that manifests itself via changes in factor shares is certainly plausible, and my results provide strong evidence that such progress is at work and a prominent source of cross-country variation in output per worker.

The remainder of the paper is organized as follows. In section 2, I disentangle physical capital's share from natural capital's share, and I disentangle human capital's share from unskilled labor's share. Estimates of factor shares are presented, and a formal analysis of the relationship between each share and output per worker is provided. In section 3, I use my factor share estimates from Section 2 and estimate the TFP residual. I then analyze the impact of allowing factor shares to vary and distinguishing between reproducible and non-reproducible factors on the variation in output per worker accruing to observables and TFP. Section 4 concludes.

2 Factor Shares and Economic Development

2.1 Theoretical Motivation

The work of Cobb and Douglas (1928) and Kaldor (1961) suggesting that factor shares were constant created a paradigm in macroeconomics. However, new theories and a general refinement in the way we think about factors and factor shares call into question the precedent set forth by Cobb and Douglas and Kaldor. Recent work in endogenous growth theory distinguishes between reproducible and non-reproducible factors and explores the idea that technical change can alter factor shares. These theoretical advances yield specific predictions about the systematic relationship between the stage of economic development and both reproducible and non-reproducible factor shares across countries.

Perpetual growth requires that the marginal products of reproducible factors of production be bounded away from zero (Jones and Manuelli, 1997). This means that the non-reproducible factors must either be augmented or eliminated. Virtually all analyses focus on augmentation. However, Peretto and Seater (2008) develop a theory of endogenous growth that focuses on factor elimination. Factor intensities are allowed to change endogenously via spending on R&D, and this serves as the catalyst for growth. As economies advance, non-reproducible factors of production become less important, and reproducible factors of production become more important. In other words, their theory predicts that non-reproducible factor intensities should decrease with output per worker, and reproducible factor intensities should increase with output per worker[2].

The Peretto and Seater theory allows for monopolistic competition in the intermediate goods sector. As a result, firms earn excess profits, and payments to the factors of production do not exhaust firm revenues. Consequently, factor intensities and factor shares, though related, are not equivalent. However, to the extent that factor shares measured using national income account data are reasonable estimates of factor intensities, the theory suggests that non-reproducible factor shares should decrease with output per worker, and reproducible factor shares should increase with output per worker.

In a related vein of the literature, Zuleta (2008b) develops an endogenous growth model in which growth occurs via capital using and labor saving technological progress. Although he incorporates endogenous factor intensities, Zuleta, unlike Peretto and Seater, does not

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[2] The term “factor intensity” refers to the elasticity of output with respect to a factor of production.

incorporate any resource absorbing activity to provide an avenue for the development of new technological knowledge. Instead, he assumes that saving can be instantaneously converted into new types of reproducible capital. Nonetheless, from an empirical standpoint, Zuleta's model yields the same testable implications pertaining to factor shares, namely that reproducible factor shares are positively related to the stage of economic development, and non-reproducible factor shares are negatively related to the stage of economic development[3].

2.2 Empirical Background

The simplest labor share calculation is computed as the fraction of real GDP attributed to employee compensation. Capital's share is then computed as the residual, $1-\left(\frac{Employee\ Compensation}{GDP}\right)$. It has been argued, most notably by Gollin (2002), that the aforementioned method, which Gollin refers to as *naïve*, is misleading because published numbers on employee compensation omit the income flowing to the self-employed. Assuming that a portion of income of the self-employed represents labor income, the consequence of this omission is estimation of labor's share that is too low and estimation of capital's share that is too high, especially in developing countries where self-employment is prevalent. Gollin adjusts for this omission by including the operating surplus of private unincorporated enterprises (*OSPUE*) in the computation of labor's share. The idea is that most self-employed people do not operate incorporated enterprises, and, consequently, capital income and labor income of the self-employed are encompassed by *OSPUE*. Gollin allocates *OSPUE* to labor and capital using three

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[3] Hansen and Prescott (2002) propose a model of transition from a primitive to an advanced economy. In their model, advancements in the stage of development, which occur via exogenous technical progress, are accompanied by decreases in land's share. Land, like other natural capital, is non-reproducible, so the prediction of this model is consistent with the aforementioned theories that suggest non-reproducible factor shares should fall with output per worker.

different adjustments and concludes that accounting for the income of the self-employed via *OSPUE* yields results indicative of stable factor shares across countries.

Gollin does not, however, perform any formal tests for correlation between either capital or labor shares and economic development. Instead, the stability claim is based on the observation that the adjustments using *OSPUE* yield capital shares that are clustered in a range from 0.15 to 0.40. Such a range, which represents almost a three-fold difference, is nontrivial, especially in the context of empirical estimation of production functions where factor shares often appear as exponents.

Using the Gollin framework, and specifically Gollin's adjustment 2, Bernanke and Gurkaynak (2001) estimate average labor shares over the period 1980-1995. They increase the number of countries for which labor shares can be calculated by constructing an *imputed OSPUE* measure. This measure is substituted in place of actual *OSPUE* for countries that report only total operating surplus and do not distinguish between the surplus of corporate enterprises and private unincorporated enterprises. Bernanke and Gurkaynak "find no systematic tendency for country labor shares to vary with real GDP per capita."

Regardless of the validity of the adjustment for self-employed income, using the standard measures of capital and labor to study the empirical relationship between factor shares and economic development is misleading if one fails to acknowledge the composite nature of the factors. Standard accounting lumps non-reproducible and reproducible factors together in composite categories. The reproducible shares need to be separated from the non-reproducible shares, and the relationship between a single factor share, not a composite share, and economic development should be analyzed.

2.3 Decomposition of Total Capital's Share

I focus first on disentangling physical capital's share from natural capital's share. Let α denote physical capital's share, and let γ denote natural capital's share. The starting point is the computation of total capital's share, $\alpha + \gamma$.

2.3.1 Total Capital's Share

I compute total capital's share according to Bernanke and Gurkaynak's variation of Gollin's adjustment 2. This computation, which is given by

$$\alpha + \gamma = 1 - \left(\frac{Employee\ Compensation}{GDP - Indirect\ Taxes - imputed\ OSPUE} \right), \tag{1}$$

is an indirect measure of total capital's share, and, specifically, it is the perfect competition counterpart to total labor's share because it is the residual remaining after total labor's share is computed and subtracted from one.

There are numerous ways to compute total capital's share and total labor's share. The approach chosen will impact the estimates of all individual shares. The entire analysis in Section 2 was also performed using two additional approaches: one that makes a similar adjustment for self-employed income and another that does not. The qualitative results are robust with respect to the treatment of self-employed income. Therefore, I relegate the results of the analysis based on the other two approaches to an Appendix, which is available upon request.

Subtracting *OSPUE* from GDP in equation (1) implies that self-employed income is dispersed between labor and capital in the same manner that corporate sector income is dispersed between the two factors. In other words, the share of labor income in *OSPUE* is assumed to be the same as the share of labor income generated in the corporate sector.

Ideally, *Indirect Taxes*, which include but are not limited to taxes on fixed assets and taxes on the total wage bill, should be allocated to capital and labor compensation depending on the tax type. However, most countries only report an aggregate tax value without any detailed breakdown of the various tax types encompassed by the aggregate value. Therefore, it is impossible to know exactly how *Indirect Taxes* should be dispersed. By subtracting *Indirect Taxes*, the implicit assumption is that the fraction of *Indirect Taxes* attributable to capital compensation is equivalent to capital's share, and the fraction of *Indirect Taxes* attributable to labor compensation is equivalent to labor's share[4].

Note that it is *imputed OSPUE* rather than *OSPUE* that enters equation (1). Though operating surplus can be broken down into corporate, unincorporated, public and private components, 1997 is the last year for which the U.N. Yearbook of National Accounts reports *OSPUE*. As is discussed later herein, data availability prevents me from disentangling physical capital's share from natural capital's share for any year except 2000. Therefore, I have to impute *OSPUE* for the year 2000, and I do so following the method of Bernanke and Gurkaynak (2001).

The *imputed OSPUE* measure is computed as the share of non-corporate employees in the labor force multiplied by private sector income. Implicit in this calculation is the assumption that the fraction of private sector income attributable to corporations is the same as the fraction of the labor force employed by corporations. Private sector income is the sum of corporate and non-corporate income, and it can also be interpreted as the sum of operating surplus and corporate employee compensation. Several different pieces of data, all of which come from

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[4] Income received by firms and not paid to owners in the form of excess profits should be paid to the factors that generate the output. Thus, for the purpose of estimating factor shares, it is misleading to treat the income received by firms and paid to the government in the form of indirect taxes as anything other than income attributed to factors of production. Doing so would skew the analysis and yield factor share estimates that account for something less than one hundred percent of factor generated income.

either the International Labor Organization's (ILO) LABORSTA database or the ILO's 2005 Yearbook of Labor Statistics, are used to perform the calculations needed to arrive at the *imputed OSPUE* measure[5].

Data for *Employee Compensation* and *Indirect Taxes* comes from table 2.3 of the 2006 version of the United Nations Yearbook of National Account Statistics. *GDP* numbers are reported in table 1.1 of the same publication.

Total capital share estimates are presented in Table 1 for the 33 countries for which the necessary data are available for the year 2000. The same shares are depicted graphically in Figure 1 where they are plotted against real GDP per worker[6]. Real GDP per worker data comes from version 6.2 of the Penn World Tables. Figure 1 suggests a quadratic relationship between total capital's share and real GDP per worker. Formal regression analysis supports this. Consider the following equation:

$$(\alpha+\gamma)_i = \psi_0 + \psi_1 u_i + \psi_2 u_i^2 + \varepsilon_i \tag{2}$$

where u_i is a coded independent variable that takes the form

$$u_i = \frac{y_i - \bar{y}}{s_y}, \tag{3}$$

ε_i is the error term, and i indexes the country. y_i is real GDP per worker in country i, and $\bar{y}$ is the average value of y in the sample. s_y is the standard deviation of the y values. The coded

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[5] First, I calculate the corporate share of the labor force by dividing *Paid Employment* by the labor force, which I compute by summing *Employment* and *Unemployment*. The share of non-corporate employees is computed as one minus the corporate share of the labor force. To obtain *imputed OSPUE*, the share of non-corporate employees is then multiplied by total corporate sector income, which is the sum of *Gross Operating Surplus* and *Employee Compensation*.

[6] The International Organization for Standardization's (ISO) three-letter country codes are used as data markers in all plots.

variable, u, is used in place of y in order to reduce the multicollinearity inherent in polynomial regression models[7].

Though OLS estimation of equation (2) reveals a negative and statistically insignificant estimate of ψ_1, an F test indicates that the quadratic model is statistically useful. The estimate of ψ_2 is positive and significant at the 5% level indicating upward concavity. The estimated slope coefficient, $\hat{\psi}_1 + 2\hat{\psi}_2 u$, is negative for lower u values and positive for higher u values. This implies that, among lower income countries, total capital's share tends to decrease as output per worker increases, and among higher income countries, total capital's share tends to increase as output per worker increases. Estimation results are reported in Table 2.

Drawing final conclusions about the relationship between total capital's share and real GDP per worker at this point would be premature. In any cross-country study, data quality is a concern. The general consensus is that the quality of economic data increases with the level of economic development. Failure to control for any systematic variation in data quality across countries could significantly impact the observed relationship between total capital's share and real GDP per worker. Specifically, if data quality is systematically related to total capital's share, then the squared residuals produced by estimation of equation (2) will fluctuate with data quality. If real GDP per worker and data quality are correlated, the squared residuals will fluctuate with real GDP per worker and introduce heteroskedasticity into the estimation of equation (2). Further precautions should be taken to ensure the observed relationship between total capital's share and real GDP per worker is representative of the actual relationship and not a

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[7] Minimizing the effects of multicollinearity is important here because multicollinearity increases the likelihood of rounding errors in the standard errors and can sometimes have an effect on the sign of regression coefficients.

mere artifact of systematic cross-country variation in data quality. I test for heteroskedasticity but find no statistical support for its presence[8].

The quadratic relationship between total capital's share and real GDP per worker, though statistically significant, is neither supported nor contradicted by economic theory. Total capital's share is an empirical measure that is often used by researchers who have intentions of estimating physical capital's share. However, as noted earlier, total capital's share is the sum of physical capital's share and natural capital's share. The aforementioned relationship is meaningful only because it suggests that physical capital's share, natural capital's share, or both are systematically related to output per worker; it is not very meaningful in and of itself. Separating physical capital's share from natural capital's share is a logical and necessary progression if the true nature of the relationship between each of these shares and the stage of economic development is to be revealed[9].

2.3.2 Physical Capital's Share

To isolate physical capital's share, I follow the approach of Caselli and Feyrer (2007). Define total wealth as the sum of physical capital and natural capital so that $W = K + N$. W is total wealth; K denotes the value of the aggregate stock of physical capital; and N denotes the value of the aggregate stock of natural capital. Like Caselli and Feyrer, I assume that differences in capital gains for natural and physical capital are negligible so that all units of wealth pay the

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[8] Heteroskedasticity is not an issue in the estimation of any of the regression equations herein, but it is an issue when estimating regression equations for which total capital shares and total labor shares are computed according to the two alternative approaches considered in the Appendix. I report White corrected *t* statistics when heteroskedasticity is detected, and I perform WLS estimation when heteroskedasticity is linked to data quality. A detailed description of how I test for heteroskedasticity and then control for it when it is present is given in the Appendix.

[9] Even if the composite relationship were insignificant, a systematic relationship between each factor share and the stage of economic development could not be ruled out. The two shares summed together may not exhibit a statistically significant correlation with the stage of economic development if a positive correlation is compensated by a negative correlation.

same return, r_w. Given this notation, total capital's share can be expressed as $\frac{r_w W}{Y}$, which, after substituting for *W*, is equivalent to $\frac{r_w(K+N)}{Y}$ where *Y* is aggregate output or GDP. This last term can be rewritten as the sum of two terms, $\frac{r_w K}{Y} + \frac{r_w N}{Y}$, the first of which is physical capital's share and the second of which is natural capital's share. Each share can be expressed as a function of total capital's share by multiplying and dividing by total wealth. Focusing for now on physical capital's share, such manipulation yields the following:

$$\frac{r_w K}{Y} = \frac{K}{W} \cdot \frac{r_w W}{Y}.$$

$$\Rightarrow \alpha = \frac{K}{W} \cdot (\alpha + \gamma) \tag{4}$$

Thus, physical capital's share is proportional to the fraction of wealth attributable to physical capital. In accordance with equation (4), estimates of α can be obtained by combining my estimates of $\alpha + \gamma$ with estimates of $\frac{K}{W}$, which can be computed using the wealth data reported in Appendix 2 of The World Bank (2006).

The World Bank splits national total wealth for the year 2000, and only the year 2000, into three components: natural capital, produced capital and intangible capital. Total wealth is estimated as the present value of future consumption. The value of the produced capital stock is computed from historical investment data using the perpetual inventory method. Natural capital is valued according to data on physical stocks of natural resources and estimates of resource rents. Intangible capital, which encompasses human capital, social capital, property rights, efficiency of the judicial system, and effectiveness of government, is measured as the residual remaining after subtracting natural and produced capital from total wealth.

Of the elements constituting intangible capital, only human capital earns income. Total capital's share does not include income paid to human capital nor the value of any other element soaked up by The World Bank's intangible capital residual. Therefore, The World Bank's total wealth measure, which includes intangible capital, is too broad and can not be used to estimate W. In addition, produced capital's value, as reported by The World Bank, encompasses the value of urban land. Land, regardless of how it is used in production, should not be interpreted as physical capital. Unlike physical capital, land can not be produced. Thus, The World Bank's estimates of produced capital's value are inappropriate estimates of K. In the context of this paper, urban land should be categorized as natural capital.

To convert the raw data provided by the World Bank into data appropriate for estimation of $\frac{K}{W}$, I proceed as Caselli and Feyrer do. First, I obtain measures of the value of the aggregate stock of physical capital, K. The World Bank follows Kunte (1998) and assumes for each country a value of urban land equal to 24 percent of the value of the aggregate stock of physical capital. So, produced capital's value equals $K + .24K$, and estimates of K are derived by dividing The World Bank's estimates of produced capital's value by 1.24. Since the value of the aggregate stock of natural capital as reported by The World Bank does not include urban land but the value of the aggregate stock of natural capital as defined herein does, it follows that urban land's value should be reallocated. To do this, I take The World Bank's estimates of produced capital's value and subtract the newly obtained estimates of K to obtain urban land values. I then add these urban land values to The World Bank's estimates of the values of the aggregate stock of natural capital to obtain corrected estimates of the values of the aggregate stock of natural capital. W is then estimated as the sum of the estimate of K and the corrected estimate of the value of the aggregate stock of natural capital. It follows that the estimate of a country's physical capital share of wealth, $\frac{K}{W}$, is computed by dividing the estimate of *K* by the estimate of W[10].

Estimates of α for the year 2000 are presented in Table 3 and plotted against real GDP per worker in Figure 2[11]. I regress α on an intercept and real GDP per worker, and OLS estimation reveals a positive and statistically significant slope coefficient at the 5% level. This indicates that physical capital's share, as predicted, is positively correlated with the stage of economic development across countries. Regression results are presented in column 1 of Table 4.

2.3.3 Natural Capital's Share

Natural capital's share can be expressed in general terms as

$$\frac{r_W N}{Y} = \frac{N}{W} \cdot \frac{r_W W}{Y}$$

$$\Rightarrow \gamma = \frac{N}{W} \cdot (\alpha + \gamma), \tag{5}$$

but given estimates of total capital's share and physical capital's share, it is easier and equivalent to back out natural capital's share as a residual. Table 5 presents the estimates of natural capital's share. These estimates are plotted against real GDP per worker in Figure 3. The scatter plot seems to indicate a negative correlation between γ and real GDP per worker, which is to be expected given the non-reproducible nature of natural capital. This is supported by OLS estimation, which indicates a negative and statistically significant relationship between the two variables at the 1% level. The regression results are reported in column 2 of Table 4.

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[10] The World Bank reports all of its data in dollars per capita.

[11] α is estimated for 31 countries. This is two fewer than the 33 for which total capital's share, $\alpha + \gamma$, was estimated. The sample is smaller because wealth data is not available for the Czech Republic and Poland.

2.4 Decomposition of Total Labor's Share

I turn now to disentangling unskilled labor's share from human capital's share. Cross country estimates of total labor's share, which are common in the literature, incorporate *Employee Compensation*. *Employee Compensation* conflates the income paid to unskilled labor and the income paid to human capital. My approach involves estimating the income paid to unskilled labor and then computing unskilled labor's share. Human capital's share is the residual left over after subtracting unskilled labor's share from total labor's share.

2.4.1 Total Labor's Share

Let η denote unskilled labor's share and let β denote human capital's share. Assuming that self-employed income is allocated to labor and capital in the same proportions as corporate sector income, total labor's share can be computed as

$$\eta + \beta = \frac{\mathit{Employee\ Compensation}}{\mathit{GDP} - \mathit{Indirect\ Taxes} - \mathit{imputed\ OSPUE}} . \tag{6}$$

The components of equation (6) and their data sources have already been discussed. Estimates of $\eta + \beta$ for 2000 are presented in Table 6 and plotted against real GDP per worker in Figure 4. The sample consists of the same 33 countries for which estimates of α, γ and $\alpha + \gamma$ were presented. The total labor share estimate is the perfect competition counterpart to the total capital share estimate. Therefore, the estimates sum to one, and statistical inference reveals a quadratic relationship between total labor's share and real GDP per worker. The only difference is that the inference for total labor's share indicates downward concavity instead of upward concavity. Nonetheless, for completeness, regression results are presented in Table 7.

2.4.2 Unskilled Labor's Share

Ashenfelter and Jurajda (2001) collect average hourly gross wage rates for McDonald's restaurants across 27 countries for the year 2000. The McDonald's rates represent different compensations for identical jobs, and the authors use the rates to perform cross-country wage comparisons[12]. I use the average McDonald's wage rate to proxy for the compensation paid to an unskilled unit of labor. Such a proxy is reasonable because the wage rates that are collected are for basic entry level jobs, and these jobs do not require experience or any type of formal education or training. Employees generally begin working as crew members and are assigned to specific food preparation stations. They are then rotated through various stations and then to the sales counter where they work as cashiers. The wages are comparable across countries because the duties performed by entry level employees are identical across countries. McDonald's restaurants operate with a standardized protocol for employee work. Food items are delivered to each restaurant in standardized freezers. The preparation of food is extremely mechanized, and the equipment used varies little across restaurants within and between countries.

Given knowledge of hours worked and the number of workers in a country, the average hourly unskilled wage rate can be converted to a total wage bill under the hypothetical scenario that all workers in a country are compensated at the unskilled wage rate. This hypothetical wage bill as a fraction of total output is my estimate of unskilled labor's share.

I obtain average hours worked per worker in the year 2000 from table 4A in the Yearly Statistics section of the ILO's LABORSTA website. This series is usually presented in terms of the average number of hours worked per week, though in a few cases, hours worked per month are reported. The type of worker encompassed by the reported averages varies from country to

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[12] McDonald's wages are different within countries and within cities. Ashenfelter and Jurajda note that these differences are usually related to full-time/part-time status and seniority. They control for both issues when compiling their data.

country. In addition, some averages are computed based on total employment, which includes employees and self-employed workers, and some are computed based only on employees.

For a few countries, average hours worked data is not reported in table 4A of the LABORSTA website. In these cases I obtain data from the ILO's October Inquiry and compute a weighted average using the number of workers employed. The October Inquiry reports average hours of work per week or per month for up to 159 occupations. Table 2B in the Yearly Statistics section of the LABORSTA database reports employment numbers categorized by industry. I weight the average hours worked for each occupation by the fraction of employees who work in the industry of which the particular occupation belongs.

To compute the total unskilled wage bill for each country in the year 2000, I first multiply the average hourly McDonald's wage rate for an individual by the average number of hours worked. I then multiply by either 52 or 12, depending on whether average hours worked is reported in per week or per month form respectively. This yields the average yearly compensation of an unskilled worker in 2000. Finally, *Employment*, which is reported in table 2A in the Yearly Statistics section of the LABORSTA database, is multiplied by average yearly compensation of an unskilled worker to obtain the total unskilled wage bill.

Two implicit assumptions associated with my approach should be noted. First, recall that average hours worked pertains to total employment for some countries and only paid employment for others. The LABORSTA database makes it clear as to which workers are included in the reported data, but when I create the average yearly compensation of an unskilled worker, I treat all average hours worked data the same. I do not distinguish between average hours worked for total employment and average hours worked for paid employment. Thus, I am assuming that average hours worked by employees is equivalent to average hours worked by the self-employed. Secondly, since *Employment* encompasses employed and self-employed workers, multiplying average yearly compensation by *Employment* means I am assuming that employed and self-employed workers command equivalent wages.

By construction, the unskilled wage bill already incorporates the labor income of unskilled self-employed workers. There is no need to make any sort of adjustment by subtracting *OSPUE*, and the unskilled wage bill is just divided by *GDP* less *Indirect Taxes* so that unskilled labor's share is given by

$$\eta = \frac{Unskilled\ Wage\ Bill}{GDP - Indirect\ Taxes}. \tag{7}$$

The data needed to estimate η is available for 15 countries, and the estimates are presented in column 1 of Table 8[13]. Figure 5 plots these estimates against real GDP per worker. OLS estimation reveals a negative relationship between unskilled labor's share and the stage of economic development. These results are presented in column 1 of Table 9, and the slope coefficient is statistically significant at the 5% level.

2.4.3 Human Capital's Share

Of the 15 countries for which η could be computed, only 10 of them overlap with countries for which $\eta + \beta$ could be computed. Column 2 of Table 8 presents the estimates of β, which are computed as residuals. Figure 6 plots these estimates against real GDP per worker. The regression results reported in column 2 of Table 9 reveal a positive slope coefficient, which

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[13] For clarity, an example of the computation of unskilled labor's share for Canada is given below. As can be seen in Table 8, unskilled labor's share in Canada is equal to 0.192. I arrive at this number in the following manner. The average hourly gross wage rate for McDonald's cashier and crew workers was equal to 6.95 Canadian dollars in 2000. Average hours worked per week by a worker in 2000, which I compute as a weighted average using the ILO's October Inquiry, is 36.9. *Employed* equals 14,764,200 in 2000. Therefore, the unskilled wage bill is equal to 6.95*36.9*52*14,764,200=$1.969\text{x}10^{11}$. GDP in Canada for the year 2000 is $1.07658\text{x}10^{12}$, and *Indirect Taxes* equal $5.1691\text{x}10^{10}$. Thus, unskilled labor's share in Canada in the year 2000 is $\frac{1.969\times10^{11}}{1.07658\times10^{12} - 5.1691\times10^{10}} = 0.192$.

is in line with theoretical predictions, but the coefficient is statistically insignificant. Thus, inference based on the 10 country full sample indicates no systematic relationship between human capital's share and the stage of economic development. However, Germany's human capital share, which takes on a value of 0.243, the lowest in the sample, is an outlier. With real GDP per worker just over $51,000, the corresponding human capital share of 0.243 stands out in Figure 6. Because there are only 10 observations, data points that take on extreme values relative to the others in the sample have a substantial impact on the OLS estimation. When Germany is omitted, the slope coefficient remains positive and becomes statistically significant at the 5% level[14].

Though this result would be more appealing had it been obtained with a larger sample, the implications of the result should not be dismissed. In spite of the small sample size, the positive correlation is confirmed statistically for real GDP per worker that ranges from about $16,600 in Russia all the way up to $67,000 in the U.S. So, the systematic relationship between human capital's share and real GDP per worker that exists when Germany is omitted is not specific to a cluster of countries at similar stages of economic development.

2.5 Remarks

The cross-country analysis of factor shares presented herein is more complete than the analyses of Zuleta (2008a) and Caselli and Feyrer (2007), and techniques that I employ represent clear departures from these studies. First, I decompose both total capital's share and total labor's share into reproducible and non-reproducible share components. Caselli and Feyrer only separate physical capital's share from natural capital's share. They do not address total labor's

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[14] The regression line shown in Figure 6 is derived after omitting Germany.

share and its components. Zuleta decomposes total capital’s share and total labor’s share, but when analyzing total capital’s share he only separates land’s share from physical capital’s share. There are other natural resources, in addition to land, that are encompassed by the typical total capital share measure. Oil, natural gas and minerals, for example, are all non-reproducible factors to which a fraction of a country’s income is paid. These additional natural resources should be distinguished from physical capital. My analysis, just as that of Caselli and Feyrer, makes this distinction and separates physical capital’s share from natural capital’s share, not just land’s share. That being said, each of the two aforementioned studies contains a crucial element that the other study omits. I incorporate elements of both studies into a single, comprehensive analysis.

Second, I control for heteroskedasticity, and, when warranted, incorporate data quality into my estimation. Although the results are unaffected, identifying and controlling for the presence of heteroskedasticity adds credibility to my approach and my inference. In any cross-country analysis, systematic variation in data quality is a concern. Knowing that the sign and significance of coefficient estimates are true reflections of the relationship between factor shares and real GDP per worker is imperative.

The most striking departure of my analysis from the current literature is the approach used to disentangle human capital’s share from unskilled labor’s share. I do not use statistical techniques or human capital proxies to obtain my share estimates. Instead, using the definition of a factor share as a guide, I combine direct observations of unskilled wage rates with employment data to obtain estimates of unskilled labor’s share. Human capital’s share is then the residual remaining after the unskilled labor share estimates are subtracted from estimates of total labor’s share.

Zuleta (2008a), who also disentangles human capital's share from unskilled labor's share, uses parameters yielded by growth regressions to obtain share estimates. The human capital proxies needed to estimate his growth regressions are computed using substantial amounts of guesswork and interpolation. The proxies are also dependent on educational attainment data that vary substantially across sources. Though my technique involves the assumption that average McDonalds' cashier and crew wages represent average unskilled labor compensation, my estimates, unlike Zuleta's estimates, are not functions of statistically estimated parameters that are subject to measurement error and dependent on the functional form of a production function.

Finally, on a much different note, I determine the significance levels of slope coefficients. Others, for whatever reason, do not perform any statistical tests to support their conclusions. Gollin (2002) and Bernanke and Gurkaynak (2001) make claims about the relationship between share estimates and output per worker by eyeballing data tables and scatter plots. I use two-tailed tests to determine the significance levels of slope coefficients for all analyses pertaining to either total labor's share or total capital's share. The purpose here is to ascertain whether there is any relationship, be it positive or negative, between share estimates and output per worker. Theory yields no predictions about the relationship, so the alternative hypothesis is that the slope coefficient differs from zero.

On the other hand, theory yields specific predictions about the nature of the relationship between non-reproducible and reproducible factor shares and output per worker. Therefore, the significance levels of slope coefficients are determined using one-tailed tests for all analyses pertaining to either physical capital's share, natural capital's share, unskilled labor's share or human capital's share. The purpose here is to ascertain whether there is a positive or negative relationship. The alternative hypothesis is that the slope coefficient is greater than zero if the factor share is reproducible and less than zero if the factor share is non-reproducible.

3 Implications for Development Accounting

The evidence presented thus far shows that factor shares, when properly defined and measured, vary systematically across countries. This suggests that factor shares should be treated as variables rather than parameters. How important is it that variation in factor shares be acknowledged when conducting empirical research? I address this question in a development accounting framework by revisiting the estimation of the TFP residual.

Let production in country *i* be characterized by

$$Y_i = A_i K_i^{\alpha_i} N_i^{\gamma_i} \left(L_i h_i - L_i\right)^{\beta_i} L_i^{\eta_i} \qquad (8)$$

where *L* is the number of workers and represents unskilled labor; *h* is a labor augmenting variable encompassing the level of education; and *A* is the TFP residual. The other variables in equation (8) have been previously defined. I take the average years of schooling for the population aged 15 and over from Barro and Lee (2001) and convert it into a proxy for human capital following Hall and Jones (1999). $h = e^{\phi(E)}$ where *E* is average years of schooling, and $\varphi(E)$ is piecewise linear with slope 0.117 for $E \leq 4$, 0.097 for $4<E\leq 8$, and 0.075 for $E>8$. The slope coefficients represent rates of return for education as reported by Psacharopoulos and Patrinos (2004). $Lh - L$ measures human capital and can be thought of as the difference between the effective workforce, which is the workforce augmented by education, and the basic workforce, which is not augmented. I use the *Economically Active Population*, which is reported in the ILO's LABORSTA database, to proxy for *L*. Data sources for all other variables are the same as the data sources used in Section 2. All data is for the year 2000.

3.1 The Impact on TFP Levels

Dividing both sides of equation (8) by *L* yields the per worker production function,

$$y_i = A_i k_i^{\alpha_i} n_i^{\gamma_i} \left(h_i - 1\right)^{\beta_i}, \tag{9}$$

where lower case letters represent per worker values. The typical development accounting approach involves the following: α_i and $\beta_i + \eta_i$ are assumed to equal to 1/3 and 2/3 respectively for all *i*; human capital and unskilled labor are assumed to be perfect substitutes; and natural capital is ignored so that γ_i equals zero for all *i*. Given equation (9), the TFP residual, *A*, can be computed in accordance with the typical approach as

$$A_i = \frac{y_i}{k_i^{1/3} h_i^{2/3}} \quad . \tag{10}$$

The exponent on physical capital per worker is 1/3, and researchers often point to this value as being consistent with the average "capital" share of national income for a broad sample of countries. But, the computations that lead to this value do not separate the income that gets paid to physical capital from the income that gets paid to natural capital. One third is the average value of total capital's share. So, not only is the systematic variation in cross-country factor shares ignored in the development accounting literature, the typical approach incorrectly assigns a factor exponent to a factor. Physical capital's share, not total capital's share, should be the exponent associated with physical capital.

Estimates of the typical TFP residual given by equation (10) are presented in Table 10 along with the two observable components of output per worker, $k^{1/3}$ and $h^{2/3}$. Notice that the TFP residual is very large relative to the *observables*. The average value of the TFP residual is 537, which is about 13 times larger than the average value of $k^{1/3}$. It is 291 times larger than the average value of $h^{2/3}$.

Including natural capital as a factor of production, treating human capital and unskilled labor as separate, imperfectly substitutable inputs, and allowing factor shares to vary yields the following TFP residual for country *i*:

$$A_i = \frac{y_i}{k_i^{\alpha_i} n_i^{\gamma_i} \left(h_i - 1\right)^{\beta_i}} . \tag{11}$$

Table 11 reports these residual values along with their observable counterparts for each country[15]. Notice that the average value of the TFP residual does not change a great deal when the typical development accounting assumptions are relaxed. In fact, statistically, the two values are equivalent; the t-statistic from a paired difference test is only equal to -0.58.

One might expect the average TFP residual to be lower in Table 11 because the residual encompasses fewer unobservable components. However, omitting natural capital and treating unskilled labor and human capital as perfect substitutes leads to an upward bias in the TFP residual that is offset by a downward bias created by the measurement error in physical capital's share[16]. When these biases are eliminated, there is very little net change in the average TFP residual.

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[15] Recall that physical capital's share and natural capital's share were computed for 31 countries. Of these 31 countries only 8 of them have the data necessary for direct computation of human capital's share. The missing human capital shares are interpolated using the intercept and slope coefficients yielded by the regression of human capital's share on real GDP per worker. Because Germany's human capital share was an outlier and omitted from the aforementioned regression, I do not include Germany in the development accounting analysis. Therefore, results in Tables 10, 11, and 12 are presented for a sample of 30 countries, not 31.

[16] There are large differences between the values of $k_i^{1/3}$ and $k_i^{\alpha_i}$. For example, when the observed value of α rather than 1/3 is inserted as the exponent on Canada's k , the value of k raised to the exponent falls from 43.96 to 6.43. This is almost a seven fold difference. On average, the value of α in the sample is smaller than 1/3, and this yields an average value of k^{α} equal to 19.60, which is roughly half the size of the average value of $k^{1/3}$, which is 40.82.

3.2 Estimating the Variation in Output per Worker accruing to *observables* and TFP

While the average TFP residual is relatively unaffected when all factors of production are acknowledged and factor shares are allowed to vary, the fraction of variation in output per worker explained by variation in the TFP residual is impacted substantially. Define $y_{observables} = k^{\alpha} n^{\gamma} (h-1)^{\beta}$ so that the per worker production function can be rewritten as $y = Ay_{observables}$. The exact form of $y_{observables}$ will change as assumptions about factors and factor shares change, but in general, the variance of output per worker can be decomposed as follows:

$$\mathrm{var}[\ln y] = \mathrm{var}[\ln A] + \mathrm{var}[\ln y_{observables}] + 2\,\mathrm{cov}[\ln A,\, \ln y_{observables}]. \tag{12}$$

How much of the variation in output per worker across countries is attributable to variation in *observables*, and how much is attributable to variation in the TFP residual? To answer this question, some assumption about the covariance must be made. One option is to ignore the covariance and assume that TFP is constant across countries. Caselli (2005) takes this approach. I find the approach unappealing because it yields relative variances that do not add up to one when the actual covariance between the TFP residual and *observables* is non-zero. Mankiw, Romer, and Weil (1992) allow TFP to vary, but given their reliance on regression analysis to obtain input measures, their covariance term is zero by construction. Their approach is just as unappealing because the correlation between *observables* and TFP is being ignored. In my sample, the correlation between TFP and *observables* equals 0.30 when the typical development accounting assumptions are made[17]. Though the relative variances are less than

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[17] The bottom of Table 12 presents all relevant variance and covariance measures, and the last row in Table 12 provides the raw correlation between *observables* and the TFP residual. As can be seen, the correlation equals 0.30 when $y = Ak^{1/3}h^{2/3}$.

one in this case, the values are still misleading[18]. Some of the variation in *observables* may actually reflect variation in TFP. Some of the variation in TFP may actually reflect variation in *observables*.

When all factors are acknowledged and factor shares are allowed to vary, TFP and *observables* are actually negatively correlated, and the relative variances are greater than one. In this case, too much of the variation in output per worker is being attributed to *observables* and too much is being attributed to TFP.

A more useful variance decomposition, which is suggested by Baier, Dwyer, and Tamura (2006)[19], is

$$\frac{\left(1-\rho_{obs.,A}^{2}\right)\mathrm{var}\left[\ln y_{observables}\right]}{\mathrm{var}[\ln y]}+\frac{\left\{sd\left[\ln A\right]+sd\left[\ln y_{observables}\right]\rho_{obs.,A}\right\}^{2}}{\mathrm{var}\left[\ln y\right]}=1. \qquad (13)$$

$\rho_{obs.,A}$ is the statistical correlation between *observables* and the TFP residual. *sd* denotes standard deviation. With this decomposition the covariance between the TFP residual and *observables* is not ignored. Rather, all of the correlation between *observables* and the TFP residual is attributed to the TFP residual. Also, the estimates of the relative variances sum to one, and interpreting each value is straightforward. The first term on the left side of equation (13) is the fraction of variation in output per worker attributable to variation in *observables*, and

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[18] When the typical development accounting assumptions are made, $\frac{\mathrm{var}\left[\ln y_{observables}\right]}{\mathrm{var}\left[\ln y\right]}$ equals 0.49. To say that 49% of income variation is explained by *observables* is misleading because implicit in such a claim is that 51% of income variation is explained by *unobservables* -- i.e., the TFP residual. This is not the case. Under typical development accounting assumptions, $\frac{\mathrm{var}\left[\ln A\right]}{\mathrm{var}\left[\ln y\right]}$ equals 0.29. So, disregarding the covariance term*,* variation in *observables* and variation in *unobservables* together explain only 78% of the variation in income. That suggests that something other than *observables* or *unobservables* explains 22% of the variation in income. Such a scenario is illogical and stems from the fact that the covariance does not equal zero.

[19] Baier, Dwyer, and Tamura (2006) use the decomposition in a growth accounting framework, but adjusting it for use in a development accounting framework is straightforward.

the second term is the fraction of variation in output per worker attributable to variation in the TFP residual[20].

Theory supports this decomposition. In the Solow model, as in the Ramsey model, the long run rate of growth equals the rate of technological progress, which is assumed to be exogenous. Variety expansion models and models of quality ladders endogenize the rate of technological progress. With the variety expansion model, technological progress occurs via an expansion of intermediate goods, which is dependent on the willingness to save, R & D costs, and the level of production technology. Quality ladder models, in addition to incorporating variety expansion, allow for increases in the quality of intermediate goods. Technological progress and the growth rate of the economy depend on the same variables that drive technological progress in variety expansion models only R & D includes the additional effort associated with improving quality.

The aforementioned theories imply that in a cross-country framework, the level of economic development is dependent on elements that are not explicitly accounted for in the production function given by equation (9). Differences in the accumulation of factors and differences in factor intensities are undoubtedly going to impact differences in output per worker, but these differences are driven by differences in saving rates, R & D costs, and production

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[20] Since it is assumed that any relationship between *observables* and the TFP residual reflects effects of the TFP residual, the covariance term along with a fraction of the variation in *observables* is added to the variance of the TFP residual so that the fraction of variation in output per worker attributable to the TFP residual can be written:

$$\frac{\mathrm{var}[\ln A] + 2\,\mathrm{cov}[\ln y_{observables}, \ln A] + \mathrm{var}[\ln y_{observables}]\rho^2_{obs.,A}}{\mathrm{var}[\ln y]}.$$

This expression is equivalent to the expression given by the second term in equation (13). The fraction of the variation in *observables* that gets allocated to the variation in the TFP residual is determined by the squared correlation, $\rho^2_{obs.,A}$. *Observables* and the TFP residual may be negatively correlated. Squaring the correlation ensures that variation in *observables* that reflects variation in the TFP residual is added to variation in the TFP residual. The fraction of variation in output per worker attributable to variation in *observables* can be written as:

$$\frac{\mathrm{var}[\ln y_{observables}] - \mathrm{var}[\ln y_{observables}]\rho^2_{obs.,A}}{\mathrm{var}[\ln y]}.$$

This expression is equivalent to the expression given by the first term in equation (13). The intuition is that any variation in *observables* that really reflects variation in the TFP residual should be attributed to variation in the TFP residual, and therefore subtracted from the variation in *observables*.

technologies, all of which are encompassed by the TFP residual. Thus, the TFP residual drives all of the variation in *observables*. Attributing all of the covariance between the TFP residual and *observables* to the TFP residual not only makes the comparison of relative variance estimates easier, it is a reasonable approach from a theoretical standpoint.

3.3 Relative Variance Estimates: Typical Assumptions

Estimates of the relative variances given by the decomposition in equation (13) are presented in Table 12 for four different combinations of assumptions pertaining to the production function. Under typical development accounting assumptions, the production function simplifies to $y = Ak^{1/3}h^{2/3}$. Given this functional form, 45% of the variation in output per worker is attributable to *observables*, and 55% is attributable to the TFP residual. This breakdown of explanatory power is consistent with the consensus view that *observables* account for at most 50% of the variation in cross-country output per worker (Caselli, 2005). This substantiates my approach because no other study that I am aware of estimates the relative variance according to equation (13). Mankiw, Romer, and Weil (1992) and Caselli (2005) ignore the covariance between the TFP residual and *observables*. Klenow and Rodriguez-Clare (1997) attribute half of the contribution of the covariance term to the TFP residual and half to *observables*[21].

3.4 Relative Variance Estimates: Allowing Factor Shares to Vary

As you move to the right in Table 12, the assumptions about the production function become increasingly consistent with reality. In the second column, factor shares are allowed to

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[21] Attributing half of the covariance term to the TFP residual and half to *observables* has no theoretical support. Klenow and Rodriguez-Clare just feel it is an "informative way of characterizing the data."

vary, but the other traditional development accounting assumptions still hold, so the production function is given by $y = Ak^{\alpha} h^{(\beta+\eta)}$. Allowing factor shares to vary has a substantial impact on the relative variance estimates. Of the variation in output per worker, 99% is now due to variation in *observables*, and only 1% is due to variation in the TFP residual. TFP's explanatory power essentially disappears under traditional development accounting if factor shares are allowed to vary.

3.4.1 Decomposing the Variation in *Observables*

This result does not indicate that variation in factor shares absorbs the shift in explanatory power. It could be that allowing factor shares to vary simply serves as an avenue for the redistribution of explanatory power to the factors. Therefore, separating the variation in output per worker explained by *observables* into that accruing to factors and that accruing to factor shares is useful. This additional breakdown of explanatory power is a two step process. First, the variation in *observables* must be broken down into the variation attributable to each of the two observable components, $\alpha \ln k$ and $(\beta + \eta) \ln h$. The second step is breaking down the variation in each observable component into that accruing to the factor and that accruing to the factor share.

The variance of *observables* can be decomposed as follows:

$$\operatorname{var}[\ln y_{observables}] = \operatorname{var}[\alpha \ln k] + \operatorname{var}[(\beta + \eta)\ln h] + 2\operatorname{cov}[\alpha \ln k, (\beta + \eta)\ln h]. \tag{14}$$

Uniquely estimating the fractions of variation in *observables* attributable to variation in $\alpha \ln k$ and variation in $(\beta + \eta)\ln h$ requires that some assumption about the covariance in equation (14) be made. No theory exists to guide this assumption. However, by considering two estimates, each of which attributes all of the correlation to either $\alpha \ln k$ or $(\beta+\eta)\ln h$, an upper and lower bound for the relative variances can be obtained.

Denote $\rho_{\alpha \ln k, (\beta+\eta)\ln h}$ as the correlation between $\alpha \ln k$ and $(\beta+\eta)\ln h$. If all of the correlation between $\alpha \ln k$ and $(\beta+\eta)\ln h$ is attributed to $\alpha \ln k$, the relative variances can be computed according to the following decomposition:

$$\frac{\left(1-\rho^2_{\alpha \ln k, (\beta+\eta)\ln h}\right)\mathrm{var}[(\beta+\eta)\ln h]}{\mathrm{var}[\ln y_{observables}]}+\frac{\left\{sd[\alpha \ln k]+sd[(\beta+\eta)\ln h]\rho_{\alpha \ln k, (\beta+\eta)\ln h}\right\}^2}{\mathrm{var}[\ln y_{observables}]}=1. \qquad (15)$$

The variation in *observables* attributable to variation in $(\beta+\eta)\ln h$ is represented by the first term on the left hand side of equation (15). The second term represents the variation in *observables* attributable to variation in $\alpha \ln k$. Alternatively, all correlation between $\alpha \ln k$ and $(\beta+\eta)\ln h$ can be attributed to $(\beta+\eta)\ln h$, in which case the relative variance decomposition takes the form:

$$\frac{\left\{sd[(\beta+\eta)\ln h]+sd[\alpha \ln k]\rho_{\alpha \ln k, (\beta+\eta)\ln h}\right\}^2}{\mathrm{var}[\ln y_{observables}]}+\frac{\left(1-\rho^2_{\alpha \ln k, (\beta+\eta)\ln h}\right)\mathrm{var}[\alpha \ln k]}{\mathrm{var}[\ln y_{observables}]}=1. \qquad (16)$$

As in equation (15), the first and second terms in equation (16) can be interpreted as the fractions of variation in *observables* attributable to $(\beta+\eta)\ln h$ and $\alpha \ln k$ respectively.

In order to break down the variation in *observables*, and ultimately the variation in output per worker, into that accruing to factors and that accruing to factor shares, the variation attributable to factors and factor shares must be extracted from the overall variation in each of the two observable components, $(\beta+\eta)\ln h$ and $\alpha \ln k$.

Focusing first on $\alpha \ln k$, let *E* denote the expectations operator and let $\Delta\alpha = \alpha - E(\alpha)$ and $\Delta \ln k = \ln k - E(\ln k)$. Following the decomposition for the variance of a product presented by Goodman (1960) and Bohrnstedt and Goldberger (1969), the variance of $\alpha \ln k$ can be written

$$\begin{aligned}\operatorname{var}[\alpha \ln k] = {} & E^2(\alpha)\operatorname{var}[\ln k] + E^2(\ln k)\operatorname{var}[\alpha] + E\left[(\Delta\alpha)^2(\Delta \ln k)^2\right] \\ & + 2E(\alpha)E\left[(\Delta\alpha)(\Delta \ln k)^2\right] + 2E(\ln k)E\left[(\Delta \ln k)(\Delta\alpha)^2\right] \\ & + 2E(\alpha)E(\ln k)\operatorname{cov}[\alpha, \ln k] - \operatorname{cov}^2[\alpha, \ln k].\end{aligned} \tag{17}$$

The first and second terms on the right hand side of equation (17) can be thought of as the direct effects of variability in $\ln k$ and α respectively. The remaining terms encompass the interaction between $\ln k$ and α. To uniquely estimate the fractions of variation in $\alpha \ln k$ accruing to α and $\ln k$, some assumption about the interaction terms must be made. Again, no theory exists to guide such an assumption, but by considering two extreme decompositions, one in which all interaction is assumed to reflect variability in α and the other in which all interaction is assumed to reflect variability in $\ln k$, the possible range of relative variance estimates can be obtained.

In the first decomposition I assume that all interaction between $\ln k$ and α reflects variability in α. The relative variance decomposition is given by

$$\frac{E^2(\ln k)\operatorname{var}[\alpha] + \mathit{Interaction}_{\alpha,\, \ln k} + E^2(\alpha)\operatorname{var}[\ln k]\rho^2_{\alpha,\, \ln k}}{\operatorname{var}[\alpha \ln k]} + \frac{\left(1 - \rho^2_{\alpha,\, \ln k}\right)E^2(\alpha)\operatorname{var}[\ln k]}{\operatorname{var}[\alpha \ln k]} = 1 \tag{18}$$

where $$\begin{aligned}\mathit{Interaction}_{\alpha,\, \ln k} = {} & E\left[(\Delta\alpha)^2(\Delta \ln k)^2\right] + 2E(\alpha)E\left[(\Delta\alpha)(\Delta \ln k)^2\right] + 2E(\ln k)E\left[(\Delta \ln k)(\Delta \ln \alpha)^2\right] \\ & + 2E(\alpha)E(\ln k)\operatorname{cov}[\alpha \ln k] - \operatorname{cov}^2[\alpha \ln k]\end{aligned}$$

and $\rho_{\alpha,\, \ln k}$ denotes the correlation between α and $\ln k$. The first term on the left hand side of equation (18) represents the fraction of variation in $\alpha \ln k$ attributable to variation in α. The second term represents the fraction of variation attributable to $\ln k$.

Alternatively, if all of the interaction is assumed to reflect variability in $\ln k$, the relative variances can be estimated according to

$$\frac{\left(1-\rho^2_{\alpha,\ln k}\right)E^2(\ln k)\mathrm{var}[\alpha]}{\mathrm{var}[\alpha\ln k]}+\frac{E^2(\alpha)\mathrm{var}[\ln k]+Interaction_{\alpha,\ln k}+E^2(\ln k)\mathrm{var}[\alpha]\rho^2_{\alpha,\ln k}}{\mathrm{var}[\alpha\ln k]}=1. \quad (19)$$

As in equation (18), the first term on the left hand side of equation (19) is the fraction of variation in $\alpha\ln k$ attributable to variation in α, and the second term is the fraction of variation in $\alpha\ln k$ attributable to variation in $\ln k$.

The variance decomposition of $(\beta+\eta)\ln h$ is identical to the decomposition given by equation (17), only $\beta+\eta$ appears in place of α, and $\ln h$ appears in place of $\ln k$. The same issue as to how the interaction terms should be treated arises, and since there is no theory for which to appeal, I follow the same methodology used with α and $\ln k$ to obtain estimates of the range of relative variances. The relative variance decompositions take the same form as those in equations (18) and (19), but $\beta+\eta$ and $\ln h$ take the place of α and $\ln k$ respectively.

Given the range of estimates for the variation in *observables* accruing to the two observable components and the range of estimates for the variation in each observable component accruing to the factor and factor share, estimates of the range of variation in output per worker accruing to each factor and factor share can be determined. For example, 99% of the variation in output per worker accrues to *observables*. Decomposing this variation in accordance with equations (15) and (16) indicates that 97-100% of the variation in *observables* accrues to $\alpha\ln k$. Of the variation in $\alpha\ln k$, the decompositions given by equations (18) and (19) reveal that 73-94% of that variation accrues to α. Therefore, the lower bound for the range of variation in output per worker accruing to α is given by the product (99%)(97%)(73%) = 70%. The upper bound for the range of variation in output per worker accruing to α is given by the product (99%)(100%)(94%) = 93%. Thus, variation in α accounts for 70-93% of the variation in output per worker. The ranges of variation in output per worker accruing to $k, \beta+\eta$, and h are determined in a similar manner. As reported in Table 12, 6-27% of the variation in output per worker accrues to k, 0-1% accrues to $\beta + \eta$, and 0-2% accrues to h.

In light of these results, it can be concluded that the variation in output per worker accrues primarily to physical capital's share. Variation in physical capital per worker absorbs the second largest fraction of variation in output per worker. Variation in total labor's share and variation in the average level of human capital augmented labor together account for a relatively small portion of the variation in output per worker. The important revelation is that the explanatory power lost by the TFP residual is not being redistributed to factors when factor shares are allowed to vary. It is the actual variation in factor shares, and primarily the variation in physical capital's share, that is absorbing TFP's lost explanatory power.

3.5 Distinguishing between Human Capital and Unskilled Labor

Though I allow factor shares to vary in the second column of Table 12, human capital and unskilled labor are still treated as perfect substitutes, and natural capital is not acknowledged. In other words, no distinction between reproducible and non-reproducible factors has been made. The per worker production function for which the results in column 3 of Table 12 are based is $y = Ak^{\alpha}(h-1)^{\beta}$, a variation of the baseline production function given by equation (9). Relative to the production function in Section 3.4, I have moved even further from the typical development accounting approach by treating human capital and unskilled labor as separate, imperfectly substitutable factors. Natural capital, however, is still omitted.

Following the decomposition given by equation (13), I find that variation in *observables* accounts for 99% of the variation in output per worker, and the remaining 1% is accounted for by variation in TFP. This breakdown is identical to that reported in column 2 of Table 12. I decompose the explanatory power of *observables* into that accruing to factors and that accruing to factor shares following the same steps described in Section 3.4.1. The only difference is that β and h-1 take the place of $\beta+\eta$ and h respectively.

The conclusions change very little. Results indicate that most of the variation in output per worker still accrues to variation in physical capital's share. Variation in physical capital per worker absorbs the majority of the remaining variation in output per worker. The labor variables, even after distinguishing between the non-reproducible factor, unskilled labor, and the reproducible factor, human capital, explain very little of the variation in output per worker.

3.6 Including Natural Capital

I use my baseline production function, $y = Ak^{\alpha} n^{\gamma} \left(h-1\right)^{\beta}$, to obtain the results in column 4 of Table 12. None of the traditional development accounting assumptions is made. All factors of production, including natural capital, are acknowledged, reproducible factors are distinguished from non-reproducible factors, and factor shares are allowed to vary. In accordance with the relative variance decomposition given by equation (13), I find that 77% of the variation in output per worker accrues to *observables*, and 23% accrues to the TFP residual. The fraction of variation accruing to *observables* decreases relative to the same fraction in columns 2 and 3 because of the relatively large magnitude of the correlation between the TFP residual and *observables*[22]. The correlation equals -0.854, and this is largely a reflection of the covariance between the TFP residual and natural capital weighted by its share in income. In columns 2 and 3, the analysis omits natural capital, and the correlation between the TFP residual and *observables* equals -0.76 and -0.78 respectively.

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[22] The intuition follows directly from equation (13). The fraction of variation in *observables* assumed to reflect variation in the TFP residual gets larger as the correlation between *observables* and the TFP residual gets larger.

I follow a two step process analogous to that described in Section 3.4.1 to decompose the explanatory power of *observables* into that accruing to factors and that accruing to factor shares. The variance of *observables*, when decomposed, can be expressed as

$$\begin{aligned}\operatorname{var}[\ln y_{observables}] = {} & \operatorname{var}[\alpha \ln k] + \operatorname{var}[\gamma \ln n] + \operatorname{var}[\beta \ln(h-1)] + 2\operatorname{cov}[\alpha \ln k, \gamma \ln n] \\ & + 2\operatorname{cov}[\alpha \ln k, \beta \ln(h-1)] + 2\operatorname{cov}[\gamma \ln n, \beta \ln(h-1)]\end{aligned} \quad . \tag{20}$$

Uniquely estimating the variation in *observables* accruing to $\alpha \ln k$, $\gamma \ln n$, and $\beta \ln(h-1)$ requires that some assumption about the covariance terms in equation (20) be made. Previously, there was only one covariance term to deal with at this stage in the process. Now there are three. However, it turns out that the last two terms in equation (20), $2\operatorname{cov}[\alpha \ln k, \beta \ln(h-1)]$ and $2\operatorname{cov}[\gamma \ln n, \beta \ln(h-1)]$, are empirically negligible. Omitting these covariances yields

$$\widetilde{\operatorname{var}}[\ln y_{observables}] = \operatorname{var}[\alpha \ln k] + \operatorname{var}[\gamma \ln n] + \operatorname{var}[\beta \ln(h-1)] + 2\operatorname{cov}[\alpha \ln k, \gamma \ln n], \tag{21}$$

which is an extremely good approximation of the actual variance. The actual variance equals 0.643 and the approximation equals 0.636.

In light of this result, I determine an upper and lower bound for the variation in *observables* accruing to each of the three components by considering two alternative relative variance decompositions. The first decomposition, which is given by

$$\frac{\left(1-\rho^2_{\alpha \ln k, \gamma \ln n}\right)\operatorname{var}[\gamma \ln n]}{\widetilde{\operatorname{var}}[\ln y_{observables}]} + \frac{\left\{sd[\alpha \ln k] + sd[\gamma \ln n]\rho_{\alpha \ln k, \gamma \ln n}\right\}^2}{\widetilde{\operatorname{var}}[\ln y_{observables}]} + \frac{\operatorname{var}[\beta \ln(h-1)]}{\widetilde{\operatorname{var}}[\ln y_{observables}]} = 1 \tag{22}$$

attributes all of the correlation between $\alpha \ln k$ and $\gamma \ln n$ to $\alpha \ln k$. $\rho_{\alpha \ln k, \gamma \ln n}$ represents the correlation between $\alpha \ln k$ and $\gamma \ln n$. The first, second, and third terms on the left hand side of equation (22) are the estimates of the variation in *observables* accruing to $\gamma \ln n$, $\alpha \ln k$, and $\beta \ln(h-1)$ respectively. If all correlation between $\alpha \ln k$ and $\gamma \ln n$ is attributed to $\gamma \ln n$, then the relative variance decomposition takes the form:

$$\frac{\left\{sd[\gamma \ln n] + sd[\alpha \ln k]\rho_{\alpha \ln k,\, \gamma \ln n}\right\}^2}{\widetilde{\mathrm{var}}[\ln y_{observables}]} + \frac{\left(1 - \rho^2_{\alpha \ln k,\, \gamma \ln n}\right)\mathrm{var}[\alpha \ln k]}{\widetilde{\mathrm{var}}[\ln y_{observables}]} + \frac{\mathrm{var}[\beta \ln(h-1)]}{\widetilde{\mathrm{var}}[\ln y_{observables}]} = 1 . \qquad (23)$$

The first, second, and third terms on the left hand side of equation (23) have the same interpretations as the corresponding terms in equation (22).

Notice that the estimate of the variation in *observables* accruing to $\beta \ln(h-1)$ is the same in both decompositions. This is because the covariance between $\beta \ln(h-1)$ and each of the other two observable components is negligible and therefore ignored[23]. The covariance between $\alpha \ln k$ and $\gamma \ln n$ is not negligible, and so the relative variance estimates for each of these components is dependent on the degree to which variation in one of the components reflects variation in the other. There is no theory suggesting that a specific fraction of the interaction between $\alpha \ln k$ and $\gamma \ln n$ be allocated to either $\alpha \ln k$ or $\gamma \ln n$. There are, however, two possible extremes: either all variation in $\alpha \ln k$ reflects variation in $\gamma \ln n$ or all variation in $\gamma \ln n$ reflects variation in $\alpha \ln k$. Thus, the relative variance estimates for $\gamma \ln n$ and $\alpha \ln k$ contained in the decompositions given by equations (22) and (23) serve as upper and lower bounds.

I break down the variation in each of the three observable components into that accruing to the factor and that accruing to the factor share as in Section 3.4.1. Equations (17), (18), and (19) pertain specifically to $\alpha \ln k$, but applying the methodology to $\beta \ln(h-1)$ and $\gamma \ln n$ is straightforward.

The break down of the explanatory power of *observables* indicates that most of the variation in output per worker accrues to physical capital's share and natural capital's share. As reported in column 4 of Table 12, 22-60% of the variation in output per worker accrues to natural

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[23] The omission of these covariances simplifies the determination of the upper and lower bound for each observable component. It is not required, though. Acknowledging all three covariances increases the number of relative variance decompositions and complicates the estimate of each relative variance without impacting either the quantitative or qualitative results in any meaningful way.

capital's share, and 10-47% accrues to physical capital's share. Variation in physical capital accounts for 1-14% of the variation in output per worker, and each of the remaining variables accounts for no more than 2% of the variation.

3.7 Acknowledging a New Type of Technical Progress

Treating factor shares as variables and acknowledging more than two factors of production has a major impact on the relative importance of TFP in explaining cross-country income differences[24]. If factor shares are treated as constant parameters, and factors of production are lumped together or omitted, variation in TFP explains a little over half of the variation in output per worker. Following the traditional accounting of factors but allowing factor shares to vary, as in column 2 of Table 12, reduces the variation in output per worker attributable to variation in TFP to 1%. If factor shares are allowed to vary and a distinction between all reproducible and non-reproducible factors is made, as in column 4 of Table 12, the variation in output per worker attributable to variation in TFP is reduced to 23%.

The key to interpreting these results is recognizing that the composition of the TFP residual changes as the assumptions about factors and factor shares change. The TFP residual is generally thought to encompass productivity and efficiency, among other things, and is often interpreted as "the" indicator of technology. But, the TFP residual also encompasses all sorts of biases and measurement errors that arise from misguided assumptions about the production process. Factor shares vary across countries, so assuming that factor shares are constant forces the actual variation in factor shares to be encompassed by the TFP residual. In addition, the omission of natural capital and the amalgamation of human capital and unskilled labor are

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[24] Caselli (2005) considers hypothetical scenarios where factor shares take on alternative constant values, but he does not allow factor shares to vary across countries nor does he distinguish between reproducible and non-reproducible factors.

misspecifications of the production function, and variation in the TFP residual will reflect these misspecifications. When factor shares are treated as variables and a distinction between all reproducible and non-reproducible factors of production is made, the TFP residual no longer encompasses the influence of factor shares and omitted or amalgamated factors on output per worker.

The typical result that the lion's share of variation in output per worker accrues to TFP is usually interpreted as evidence of technology's importance in explaining cross-country differences in output per worker. The evidence presented here reveals that the overwhelming majority of cross-country variation in output per worker accrues to factor shares, not TFP. In principle, a change in technology could manifest itself as a change in any of the production function's parameters. Changes in technology are not synonymous with changes in TFP. TFP is a residual and picks up everything, including technical progress, not explicitly accounted for by the production function. However, TFP enters the production function in a linear fashion, and so only technical progress of a factor augmenting nature is appropriately accounted for by the TFP residual.

There is no reason to believe that technical progress can not manifest itself as a change in factor shares. In fact, there is a theoretical precedent for such progress. Peretto and Seater (2008) and Zuleta (2008b) develop endogenous growth models whereby technical progress occurs via changes in factor shares[25]. This type of progress impacts the intensity with which factors of production are used. It does not impact the effectiveness or productivity of factors of production, and so it is fundamentally different from factor-augmenting technical progress.

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[25] These theories pertain specifically to factor intensities. To the extent that factor shares reasonably approximate factor intensities, the theories imply that technical progress occurs via changes in factor shares. Peretto and Seater (2008) refer to this type of technical progress as factor eliminating technical progress.

If factor shares are assumed constant, technical progress that impacts factor shares, in addition to factor-augmenting technical progress, is absorbed by the TFP residual. Two different types of technical change, only one of which is correctly accounted for by the multiplicative residual parameter, are being entangled and forced to work through the same channel. Consequently, variation in the TFP residual inaccurately reflects variation in technical progress, and technical progress' role in explaining cross-country variation in output per worker can not be accurately gauged.

The explanatory power of the TFP residual is reduced by a substantial margin when factor shares are allowed to vary. The evidence, however, does not diminish the role of technical change. Rather, the evidence indicates the importance of acknowledging a new type of technical change, one that affects factor shares.

4 Conclusion

Skepticism about the constancy of factor shares dates back to the time of Keynes and Solow, but only recently have theoretical analyses like that of Peretto and Seater (2008) and Zuleta (2008b) yielded specific predictions about the systematic relationship between cross-country factor shares and the stage of economic development. I provide empirical evidence consistent with these theoretical claims, and, specifically, my results reveal that non-reproducible factor shares decrease with the stage of economic development, and reproducible factor shares increase with the stage of economic development. This result suggests that factor eliminating technical progress is a potentially important phenomenon, and incorporation of such progress into models of economic growth should be considered.

In addition, theoretical or empirical studies that incorporate the assumption of constant factor shares should be revisited. Researchers rarely make a distinction between reproducible and non-reproducible factors. As a result, the shares that are typically considered are composite shares that conflate the fractions of income paid to fundamentally different factors of production. A very common approach is to combine all factors of production into one of two categories: capital or labor. The standard capital share measure conflates physical capital's share and natural capital's share. The standard labor share measure conflates human capital's share and unskilled labor's share. Failure to acknowledge the composite nature of the standard share measures can yield misleading conclusions. The results presented herein reveal that the systematic relationship between composite shares and the stage of economic development is different from the systematic relationship between a single, non-reproducible or reproducible share and the stage of economic development. Kaldor (1961), whose "stylized facts" are often cited, concluded that factor shares were constant over time and across countries without making a distinction between reproducible and non-reproducible factors. This distinction turns out to be very important.

In the second part of the paper, I revisit the development accounting exercise, acknowledging variation in factor shares and making a distinction between non-reproducible and reproducible factors. The goal of development accounting is to explain cross-country differences in output per worker. Though the TFP residual is generally thought to proxy for technology, technical progress can arrive in many forms, not all of which are correctly accounted for by a multiplicative factor that enters the production function linearly. Moreover, TFP is a residual and is more accurately described as a "measure of our ignorance." The variation in output per worker attributable to variation in TFP is really variation in output per worker that is unexplained.

The general consensus is that at least half of the variation in output per worker is attributable to variation in TFP. Researchers have attempted a number of things in an effort to chip away at TFP's explanatory power. Caselli (2005) takes inventory of these attempts, which include: improvements in the accuracy of human and physical capital measures; acknowledgement of embodied technical progress; acknowledgement of the prevalence of agriculture in developing countries; and consideration of non-neutral differences in technology. However, the result that the majority of the variation in output per worker accrues to the TFP residual is robust to the aforementioned efforts. In fact, no improvements in measurement or methodology have led to a substantial reduction in the importance of the TFP residual until now.

Other development accounting analyses assume factor shares are constant and make no distinction between reproducible and non-reproducible factors. My approach yields results that stand in stark contrast to those previously attained. When factor shares are allowed to vary and a distinction between reproducible and non-reproducible factors is made, the majority of variation in output per worker accrues to factor shares, specifically physical capital's share and natural capital's share. TFP's explanatory power decreases by more than 30 percentage points.

This decrease in explanatory power does not imply a decrease in the importance of technical progress. Not all technical progress is accurately accounted for by the TFP residual. For example, technology that is embodied in new physical capital is fundamentally different from technology that augments factors. Denison (1962), Griliches (1963), and Jorgenson and Griliches (1967) recognized the failure of TFP to properly account for "embodied" technical change, and argued that the TFP residual played a smaller role in explaining variation in output per worker than previously thought.

In similar spirit, I submit that factor eliminating technical progress, which impacts factor shares, is fundamentally different from factor augmenting technical progress, and therefore improperly accounted for by the TFP residual. By assuming factor shares are constant, factor eliminating technical progress is forced to work through the TFP residual. As a result, the variation in TFP misrepresents the variation in factor eliminating technical progress.

The shift in explanatory power from the TFP residual to factor shares that occurs when factor shares are allowed to vary does not diminish the role of technical progress. There is no reason to believe that variation in factor shares and variation in technical progress are independent of each other. To the extent that technology is embodied in factor shares, variation in factor shares reflects variation in technical progress. Identifying and understanding the determinants of cross-country variation in factor shares is imperative to understanding cross-country differences in output per worker.

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Table 1

Total Capital's Share, 2000

Country	Total Capital's Share	Country	Total Capital's Share
Australia	0.384	Japan	0.256
Austria	0.398	Korea, Republic Of	0.332
Belgium	0.340	Mauritius	0.354
Botswana	0.534	Mexico	0.518
Canada	0.334	Netherlands	0.418
Costa Rica	0.345	New Zealand	0.418
Czech Republic	0.472	Norway	0.526
Denmark	0.408	Panama	0.361
Egypt	0.538	Poland	0.379
Finland	0.418	Portugal	0.326
France	0.376	Russia	0.485
Germany	0.360	Singapore	0.443
Greece	0.443	Spain	0.306
Hungary	0.400	Sweden	0.351
Ireland	0.497	Trinidad and Tobago	0.409
Israel	0.313	U.S.A	0.320
Italy	0.408		

Source : Author's Calculations.

Table 2

Total Capital's Share

Variable	
Intercept	0.361***
	(20.886)
u	-0.011
	(-0.910)
u^2	0.04***
	(2.890)
F-test for overall significance of regression	5.735
	[3.316]
Adjusted R^2	0.228
F-test for no heteroskedasticity	3.215
	[3.316]
Sample	33 obs.

--Dependent variable is Total Capital's share.

--*u* is a coded independent variable used in place of real GDP per worker.

--t-statistics are in parantheses.

--brackets are 5% critical values of the F distribution.

--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Table 3

Physical Capital's Share, 2000

Country	Physical Capital's Share	Country	Physical Capital's Share
Australia	0.219	Japan	0.204
Austria	0.293	Korea, Republic Of	0.251
Belgium	0.261	Mauritius	0.271
Botswana	0.318	Mexico	0.289
Canada	0.164	Netherlands	0.305
Costa Rica	0.137	New Zealand	0.154
Denmark	0.287	Norway	0.291
Egypt	0.237	Panama	0.200
Finland	0.284	Portugal	0.236
France	0.273	Russia	0.186
Germany	0.273	Singapore	0.357
Greece	0.309	Spain	0.222
Hungary	0.245	Sweden	0.249
Ireland	0.327	Trinidad and Tobago	0.105
Israel	0.231	U.S.A	0.218
Italy	0.302		

Source : Author's Calculations.

Table 4

Physical Capital's Share and Natural Capital's Share

	Dependent Variable	
Variable	Physical Capital's Share	Natural Capital's Share
Intercept	0.200***	0.251***
	(6.880)	(7.656)
real GDP per worker, y	1.162E-06**	-2.421E-06***
	(1.791)	(-3.307)
Adjusted R^2	0.069	0.249
F-test for no heteroskedasticity	0.511	0.205
	[3.340]	[3.340]
Sample	31 obs.	31 obs.

--t-statistics are in parantheses.

--brackets are 5% critical values of the F distribution.

--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Table 5

Natural Capital's Share, 2000

Country	Natural Capital's Share	Country	Natural Capital's Share
Australia	0.165	Japan	0.052
Austria	0.106	Korea, Republic Of	0.080
Belgium	0.079	Mauritius	0.083
Botswana	0.217	Mexico	0.230
Canada	0.170	Netherlands	0.114
Costa Rica	0.207	New Zealand	0.264
Denmark	0.121	Norway	0.235
Egypt	0.301	Panama	0.161
Finland	0.134	Portugal	0.091
France	0.103	Russia	0.299
Germany	0.087	Singapore	0.086
Greece	0.134	Spain	0.084
Hungary	0.156	Sweden	0.102
Ireland	0.170	Trinidad and Tobago	0.304
Israel	0.081	U.S.A	0.102
Italy	0.106		

Source : Author's Calculations.

Table 6

Total Labor's Share, 2000

Country	Total Labor's Share	Country	Total Labor's Share
Australia	0.616	Japan	0.744
Austria	0.602	Korea, Republic Of	0.668
Belgium	0.660	Mauritius	0.646
Botswana	0.466	Mexico	0.482
Canada	0.666	Netherlands	0.582
Costa Rica	0.655	New Zealand	0.582
Czech Republic	0.528	Norway	0.474
Denmark	0.592	Panama	0.639
Egypt	0.462	Poland	0.621
Finland	0.582	Portugal	0.674
France	0.624	Russia	0.515
Germany	0.640	Singapore	0.557
Greece	0.557	Spain	0.694
Hungary	0.600	Sweden	0.649
Ireland	0.503	Trinidad and Tobago	0.591
Israel	0.687	U.S.A	0.680
Italy	0.592		

Source : Author's Calculations.

Table 7

Total Labor's Share

Variable	
Intercept	0.639***
	(36.995)
u	0.011
	(0.910)
u^2	-0.040***
	(-2.890)
F-test for overall significance of regression	5.735
	[3.316]
Adjusted R^2	0.228
F-test for no heteroskedasticity	3.215
	[3.316]
Sample	33 obs.

--Dependent variable is Total Labor's share.

--*u* is a coded independent variable used in place of real GDP per worker.

--t-statistics are in parantheses.

--brackets are 5% critical values of the F distribution.

--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Table 8

Unskilled Labor's Share and Human Capital's Share, 2000

Country	Unskilled Labor's Share	Human Capital's Share
Brazil	0.207	
Canada	0.192	0.474
Czech Republic	0.207	0.321
Germany	0.396	0.243
Hong Kong	0.086	
Japan	0.261	0.483
Korea, Republic Of	0.195	0.473
Philippines	0.500	
Poland	0.206	0.415
Russia	0.252	0.263
Singapore	0.141	0.416
Sweden	0.204	0.445
Thailand	0.410	
UK	0.241	
USA	0.172	0.508

Source: Author's Calculations.

Table 9

Unskilled Labor's Share and Human Capital's Share

	Dependent Variable		
	Unskilled Labor's Share	Human Capital's Share	
Variable			Omit Germany
Intercept	0.347***	0.313***	0.302***
	(6.197)	(4.076)	(5.683)
real GDP per worker, y	-2.840E-06**	2.247E-06	3.049E-06**
	(-2.056)	(1.286)	(2.474)
Adjusted R^2	0.187	0.068	0.390
F-test for no heteroskedasticity	0.497	0.172	2.805
	[3.885]	[4.737]	[5.143]
Sample	15 obs.	10 obs.	9 obs.

--t-statistics are in parantheses.

--brackets are 5% critical values of the F distribution.

--*indicates significance at the 10% level, ** at the 5% level, and *** at the 1% level.

Table 10

Decomposition of Output per Worker
Factor Shares Constant; Natural Capital Omitted

Country	y	$k^{1/3}$	$h^{2/3}$	A
U.S.A	67078.860	50.530	2.167	612.705
Norway	63909.140	56.919	2.145	523.442
Belgium	59873.550	49.137	1.892	644.007
Ireland	59103.420	43.348	1.893	720.257
Singapore	58750.040	48.878	1.664	722.335
Austria	58441.050	49.415	1.801	656.782
Netherlands	56690.570	46.328	1.893	646.416
France	55285.960	47.132	1.753	668.955
Israel	51882.640	45.138	1.917	599.646
Italy	50853.040	46.704	1.678	648.860
Australia	50606.350	45.300	2.048	545.588
Denmark	50448.300	49.464	1.923	530.483
Canada	49815.630	43.958	2.121	534.426
Sweden	46544.490	45.732	2.098	485.022
Finland	45192.140	46.045	1.955	502.147
Japan	44563.230	61.015	1.904	383.513
Spain	44360.540	41.548	1.689	632.168
New Zealand	40976.960	39.054	2.133	491.845
Mauritius	34617.690	27.912	1.555	797.702
Portugal	34000.270	36.699	1.542	600.917
Trinidad and Tobago	33101.830	29.732	1.742	639.045
Greece	32069.690	38.038	1.830	460.776
Korea, Republic Of	30620.650	37.850	2.039	396.677
Hungary	23788.820	31.222	1.871	407.151
Costa Rica	20596.220	26.548	1.560	497.370
Mexico	19621.490	35.444	1.683	328.835
Panama	18798.390	27.108	1.819	381.275
Russia	17269.690	29.281	1.958	301.150
Botswana	16616.550	27.637	1.583	379.771
Egypt	11939.540	21.579	1.506	367.322
Average	41580.558	40.823	1.845	536.886
Standard Deviation	16081.332	9.777	0.196	129.625
correlation with y (logs)	1	0.894	0.497	0.745
correlation with A (logs)	0.745	0.409	-0.073	1

Table 11

Decomposition of Output per Worker
Factor Shares Vary; Natural Capital Included; Labor Inputs Separated

Country	y	k^{α}	n^{γ}	$(h-1)^{\beta}$	A
U.S.A	67078.860	12.961	3.079	1.489	1129.164
Norway	63909.140	34.127	16.494	1.448	78.431
Belgium	59873.550	21.099	2.286	1.252	991.572
Ireland	59103.420	40.423	6.141	1.252	190.203
Singapore	58750.040	64.537	2.406	1.059	357.494
Austria	58441.050	30.685	3.098	1.179	521.419
Netherlands	56690.570	33.269	3.314	1.249	411.772
France	55285.960	23.567	2.966	1.139	694.406
Israel	51882.640	14.058	2.331	1.260	1256.890
Italy	50853.040	32.565	3.046	1.076	476.441
Australia	50606.35	12.210	6.315	1.350	486.217
Denmark	50448.300	28.832	3.717	1.262	373.099
Canada	49815.630	6.425	6.899	1.418	792.503
Sweden	46544.490	17.396	2.936	1.374	663.429
Finland	45192.140	26.134	4.225	1.276	320.660
Japan	44563.230	12.424	1.760	1.265	1610.919
Spain	44360.540	12.021	2.354	1.082	1448.755
New Zealand	40976.960	5.417	21.083	1.385	259.040
Mauritius	34617.690	14.923	2.086	0.974	1141.764
Portugal	34000.270	12.749	2.444	0.963	1132.773
Trinidad and Tobago	33101.830	2.913	30.400	1.116	335.087
Greece	32069.690	29.080	3.872	1.175	242.374
Korea, Republic Of	30620.650	15.489	2.191	1.359	663.929
Hungary	23788.820	12.491	4.647	1.193	343.559
Costa Rica	20596.220	3.867	8.362	0.979	650.261
Mexico	19621.490	21.984	11.086	1.068	75.402
Panama	18798.390	7.219	4.777	1.155	472.087
Russia	17269.690	6.575	23.876	1.157	95.079
Botswana	16616.550	23.640	7.963	0.997	88.535
Egypt	11939.540	8.853	17.302	0.941	82.828
Average	41580.558	19.598	7.115	1.196	579.536
Standard Deviation	16081.332	13.285	7.336	0.153	430.812
correlation with y (logs)	1	0.465	-0.461	0.594	0.501
correlation with A (logs)	0.501	-0.128	-0.740	0.160	1

Table 12

Decomposing the Variability in Output per Worker

Variance Decomposition	Production Function			
	$y=Ak^{1/3}h^{2/3}$	$y=Ak^{\alpha}h^{\beta+\eta}$	$y=Ak^{\alpha}(h-1)^{\beta}$	$y=Ak^{\alpha}n^{\gamma}(h-1)^{\beta}$
Variation in Output per Worker attributable to *Observables*	0.45	0.99	0.99	0.77
Variation accruing to α		0.7- 0.93	0.67 - 0.90	0.10 - 0.47
Variation accruing to k		0.06 - 0.27	0.06 - 0.27	0.01 - 0.14
Variation accruing to $\beta+\eta$		0 - 0.01		
Variation accruing to β			0 - 0.02	0 - 0.01
Variation accruing h		0 - 0.02		
Variation accruing to h-1			0.02 - 0.06	0.01 - 0.02
Variation accruing to γ				0.22 - 0.60
Variation accruing to n				0 - 0.08
Variation in Output per Worker attributable to the TFP residual	0.55	0.01	0.01	0.23
Variances and Covariances				
var(ln(y))	0.224	0.224	0.224	0.224
var(ln(A))	0.065	0.355	0.391	0.828
var(α ln(k))	0.066	0.529	0.529	0.529
var($\beta+\eta$ (ln(h)))	0.012	0.016		
var(β ln(h-1))			0.017	0.017
var(γ ln(n))				0.656
cov[ln(A), α ln(k)]	0.027	-0.355	-0.376	-0.093
cov[ln(A), ($\beta+\eta$)ln(h)]	-0.002	0.031		
cov[ln(A), β ln(h-1)]			0.012	0.016
cov[ln(A), γ ln(n)]				-0.546
cov[α ln(k), γ ln(n)]				-0.283
cov[αln(k), ($\beta+\eta$)(ln(h))]	0.016	-0.014		
cov[αln(k), β (ln(h-1))]			0.007	0.007
cov[γ ln(n), β (ln(h-1))]				-0.004
Raw Correlation				
correlation coefficient, $\rho_{obs,A}$	0.30	-0.76	-0.78	-0.85

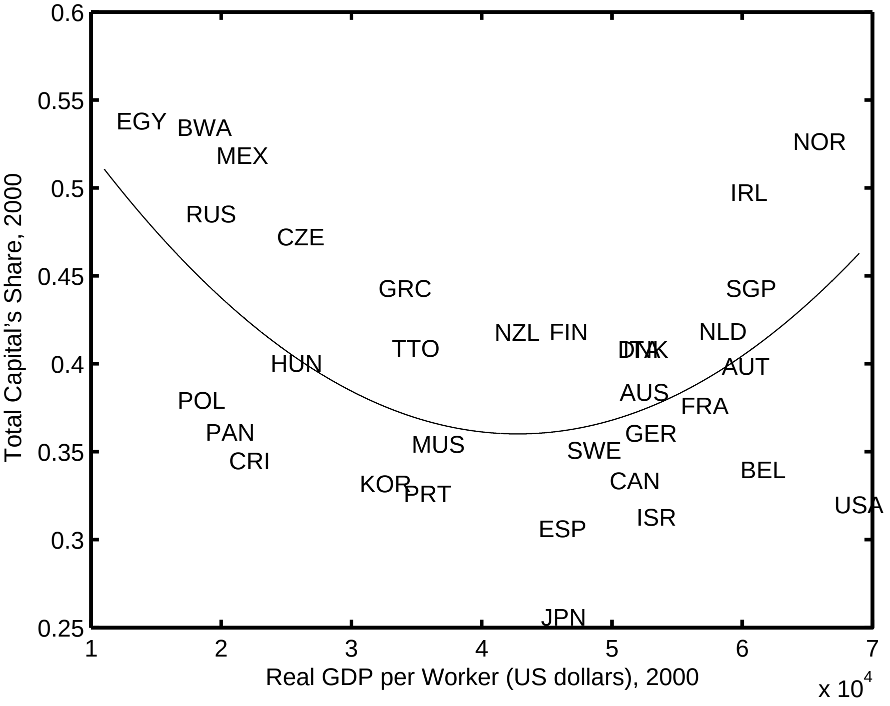

Figure 1—Total Capital's Share vs. Real GDP per Worker

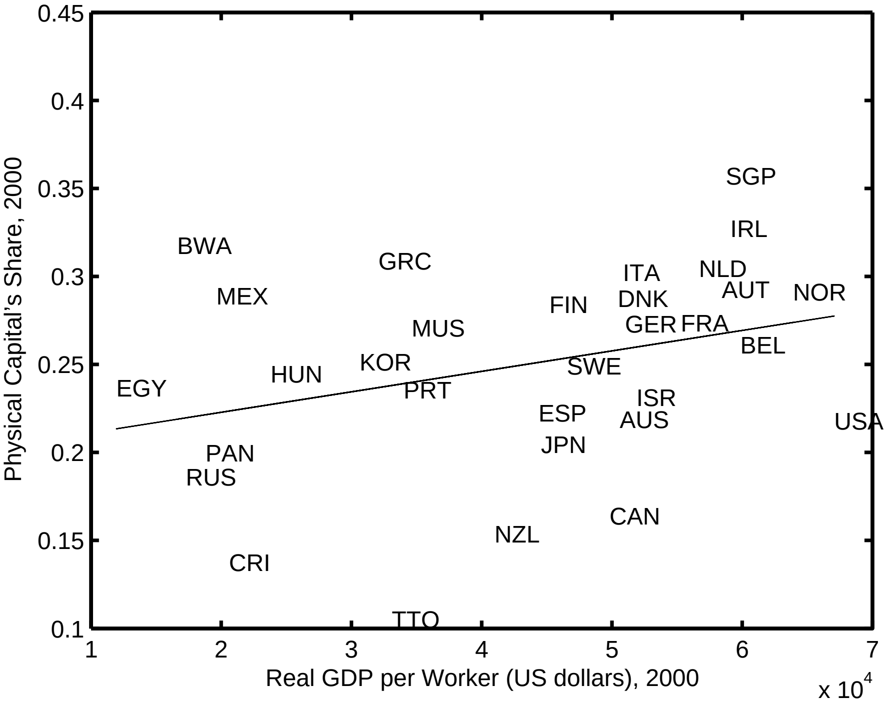

Figure 2—Physical Capital's Share vs. Real GDP per Worker

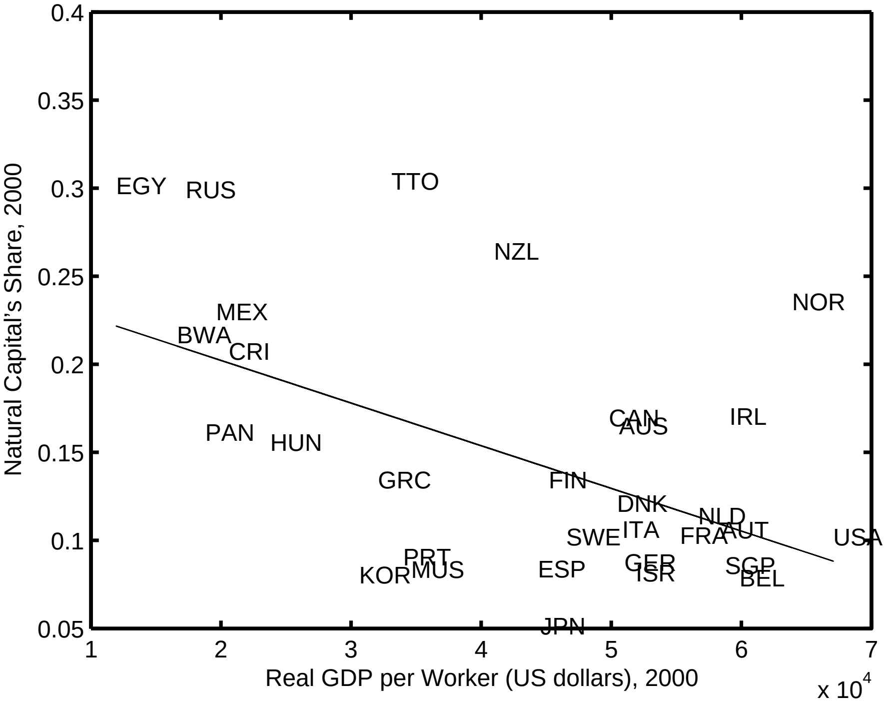

Figure 3—Natural Capital's Share vs. Real GDP per Worker

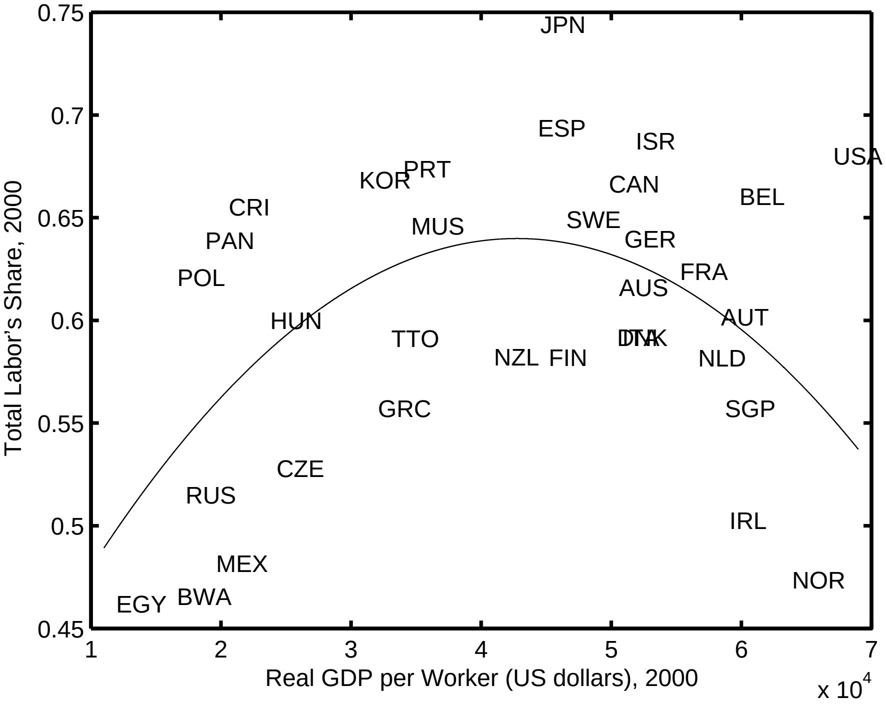

Figure 4—Total Labor's Share vs. Real GDP per Worker

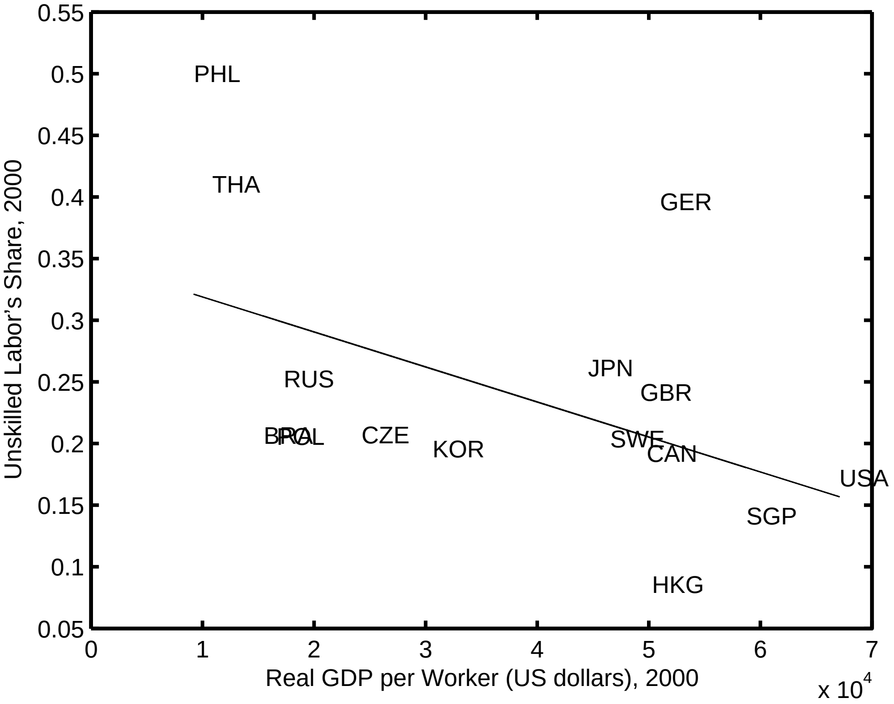

Figure 5—Unskilled Labor's Share vs. Real GDP per Worker

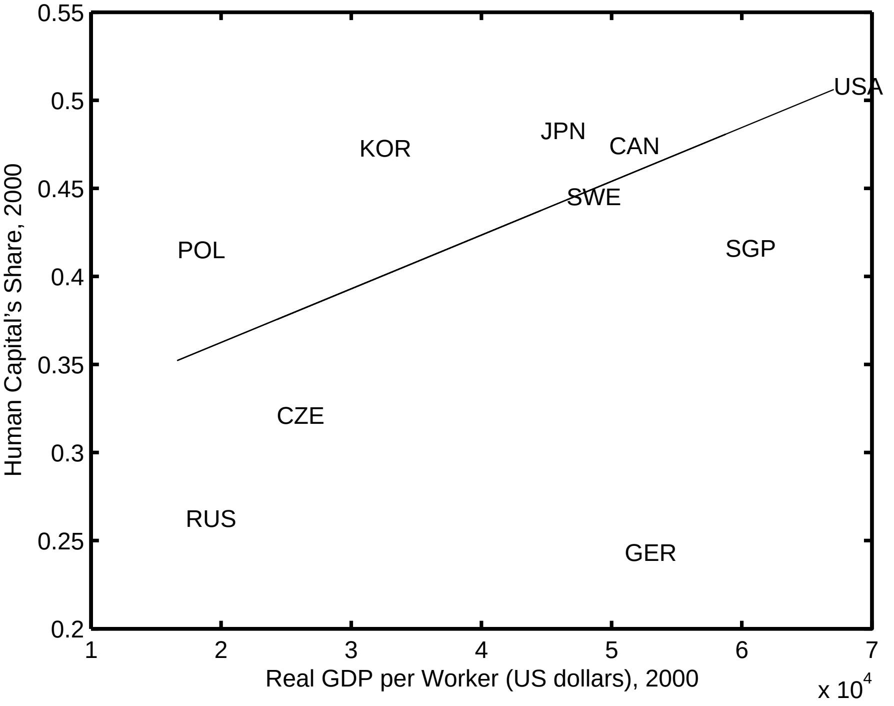

Figure 6—Human Capital's Share vs. Real GDP per Worker