

Stochastic Dominance, Entropy and Biodiversity Management

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Abstract. In this paper we develop a model of population dynamics using the Shannon entropy index, a measure of diversity that allows for global and specific population shocks. We model the effects of increasing the number of parcels on biodiversity, varying the number of spatially diverse parcels to capture risk diversification. We discuss the concepts of stochastic dominance as a means of project selection, in order to model biodiversity returns and risks. Using a Monte Carlo simulation we find that stochastic dominance may be a useful theoretical construct for project selections but it is unable to rank every case.

*“In the world of biology, as in the world of finance, diversity is our only hedge against unknown and future risks.”* John Madson

## **1. Introduction**

Conservation organizations, such as the Nature Conservancy, play an important role in the preservation of biodiversity. According to the Nature Conservancy’s mission statement, its goal is: *To preserve plants, animals and natural communities that represent the diversity of life on Earth by protecting the lands and waters they need to survive.* This type of mission statement implies that the focus of conservation organizations is not solely on the ranking of preservation projects in terms of returns to biodiversity but also the consideration of those projects in terms of risks to biodiversity. When ranking sites for potential preservation, the Nature Conservancy ranks the species or communities present based on their rarity (from “critically important” to “demonstrably secure”) and also ranks the likelihood that the species or community will survive on that site if it is preserved (from highly likely to highly unlikely) (see Weitzman 1995).

The selection of the preservation projects is difficult due to the uncertainty of the biodiversity associated with individual parcels, the interaction between different preserves and the threats to preserved areas. There are numerous threats to biodiversity, including demographic stochasticity, global warming, political uncertainty, poor enforcement of property rights, genetic deterioration and natural catastrophes.

The economic threats to biodiversity fall into two categories, those related to market failure and those related to government failure. The threats related to market failure are those that arise when urban, agricultural, forestry, and other uses of land threaten habitat that promotes

biodiversity. Those who pursue development of habitat have little incentive to consider the positive impacts of biodiversity on private land. In the case of forestry, for example, logging of old growth forests has threatened the spotted owl (Rubin, Helfand, Loomis 1991; Hagan, Vincent, and Welle 1992). Longleaf pine ecosystems are economically disadvantaged relative to slash pine forests since their amenity benefits are not considered by private landowners (Alavalapati, Stainback, and Carter 2002). Beach erosion as a result of beach hardening has threatened sea turtle nesting habitat (Whitehead 1993). Urbanization and hunting have threatened wolves (Chambers and Whitehead 2003). Numerous studies have measured the benefits of preservation of these and other species. Loomis and White (1996) review a number of such studies measuring the external market benefits of specific threatened fish and wildlife species. They argue that valuation studies should focus on the benefits of preserving ecosystems and not individual species. Even in the case of anthropocentric valuation, in which humans are concerned only about the benefits they receive from a specific species, "...ecological inter-relationships necessary to support the high-profile species may mean that the entire ecosystem must be protected."

Perverse incentives inherent in the Endangered Species Act (ESA) produce a major source of government failure as a threat to biodiversity preservation. There is a lack of information about the value of biodiversity on private land. Since designation of an endangered species lowers private property values due to restrictions on development, there is little incentive for landowners to cooperate with regulators who wish to examine the private property (Polasky, Doremus and Rettig 1997). This lack of cooperation limits the global knowledge about endangered species, creates uncertainty about the benefits of preservation, and lowers the activities related to preservation for risk averse regulators.

Another threat to biodiversity related to the ESA is anti-conservation investment (Michael and Lueck 2003). Regulated private property is not subject to compensation. In order to avoid future regulation under the ESA, a landowner has an incentive to strategically develop their private property before the presence of an endangered species is recognized by regulators. The development eliminates the species from the property. Without the threat of regulation, the optimal time for development is farther into the future, implying that the endangered species would be preserved on the private property for a longer time period, possibly allowing greater recovery of the population.

A related threat to biodiversity is the distribution of benefits and costs associated with species preservation (Brown and Shogren 1998). The benefits of species preservation are dispersed over the entire human population. The high individual costs are concentrated on those landowners and business firms that are negatively impacted by preservation (Montgomery, Brown, and Darius 1994; Innes, Polasky, and Tschirhart 1998). Even if the aggregate benefits are greater than the aggregate costs in a public choice framework, lobbying efforts by those with concentrated costs have greater political clout than the quiet masses with relatively low individual benefits. Government budget constraints limit the number of species that can be listed (Brown and Shogren 1998). There are cases where high benefits are concentrated in the hands of a few individuals or corporations. For example, biodiversity has value to the pharmaceutical industry. However, even in this case, the marginal benefits of protecting any single plant or animal species are low relative to the marginal costs due to the potential for species substitution. Therefore, there is little private incentive to preserve biodiversity.

In the past, environmental policies to protect endangered species have focused on economic uncertainty (e.g., the uncertainty of future costs and benefits) while largely ignoring

uncertainty arising from environmental factors: natural disasters such as wildfires or hurricanes that destroy habitat, global warming, genetic deterioration, and demographic stochasticity. A greater margin of safety is required if such policies are to be successful in maintaining endangered populations. Obviously, increasing the total size of the population provides protection, since a smaller population has a greater probability of varying to zero in the face of a disaster or demographic stochasticity. Another way to provide additional safety is to increase the geographic range of the targeted species. This could be done by preserving larger areas, or through the use of redundant preserves, i.e., preserving several geographically diverse areas with similar habitats. This will decrease the probability that the entire population will be struck by the same environmental problem (Noss and Cooperrider 1994).

Ludwig, Hilborn and Walters (1993) illustrate the results of failing to take uncertainty into account in the case of an exploitable resource. Governments regularly regulate harvests of such resources in order to achieve a sustainable level of exploitation (maximum sustained yield). These authors suggest that this ideal has never, and perhaps can never, be reached. There is not, and we can never expect to achieve, scientific consensus as to what this sustainable harvest rate will be. Political and economic pressures often force greater, rather than smaller, harvests. The populations of harvested species experience natural fluctuation. During strong periods, there is greater investment and increased harvests. When populations return to normal or below normal levels, disinvestment is too politically costly. The results are high subsidies and persistent over-harvesting. Because each system is unique, it is extremely difficult to predict the effects of exploitation. This inability to predict should lead to more caution than is the norm. What is needed are actions robust to the inevitable uncertainties and a degree of hedging in environmental policy.

As John Madson implies, the uncertainty of the biological world resembles the financial world in many ways. Using this concept as a starting point, we apply financial strategies of diversification and stochastic dominance to the process of conserving biodiversity. We proceed as follows. In section 2, we discuss the difficulties of measuring the returns from biodiversity. We then develop a model of population dynamics using the Shannon entropy index as a measure of diversity. Specifically, we develop a model of species dynamics which allows for global and specific population shocks. In section 3, we model the effects of increasing the number of parcels on biodiversity and provide an argument for spatially diverse parcels. Within this section, we vary the number of spatially diverse parcels to capture the potential risk diversification effects. Next, in section 4, we discuss the concepts of stochastic dominance as a means of project selection, in order to model biodiversity returns and risks. A Monte Carlo experiment is used to perform simulations for ten parcels with 100 distinct species that may inhabit each parcel. We find that stochastic dominance may be a useful theoretical construct to rank parcels but it is unable to rank every case. In the final section, we provide some policy implications and the direction of our future work.

## **2. Biodiversity, Dynamics and Returns**

The interaction of parcels and the potential for risk reductions have clear policy implications for conservation organizations. Conservation organizations range from small grassroots organizations to the Nature Conservancy, which is the largest private owner of land in the United States. Within this section, we argue that large conservation organizations face less risk in maintaining biodiversity than a relatively small, highly concentrated local organization. The advantage of the large conservation organization arises from the tendency to have many

spatially diverse projects. Although organizations, such as the Nature Conservancy, may maintain many different preserves to attract potential members and influence the level of donations, its members are better off from this diversification. Specifically, the members benefit because a large number of spatially diverse preserves significantly reduces the risk to the overall level of biodiversity. Alternatively, a number of smaller organizations may cooperate in order to ensure that a diverse range of projects is preserved. The smaller organizations may exist since they may serve to provide a regional focus or a niche focus. By cooperating unintentionally or deliberately, the group of smaller organizations mimics a single, large organization. In either scenario, there exists a spillover public good, in that the risk to biodiversity is reduced through either a large single organization or the holdings of numerous smaller organizations. To illustrate the effects of increasing the number of preserves on the overall risk, measured in terms of variance of biodiversity, we allow the number of equal sized parcels to increase. There exists a portion of the risk (global shocks) to biodiversity that is common or systematic to all parcels and this risk is unaffected by the number of parcels.

In the financial economics literature, diversification is relatively easy to apply since financial returns are well defined. However, this is not the case for the returns to the conservation organization whose goal is to maximize biodiversity. There are two primary indices used to measure species abundance. The simplest is the Simpson index,  $SP = 1/\sum(p_i)^2$ , where  $p_i$  represents the proportion of each of  $i$  species in the total sample (Simpson, 1949). The Shannon-Weaver index is calculated as  $SW = -\sum(p_i \log(p_i))$ . Although both indices are widely used, they may behave slightly differently. The Simpson index tends to be more sensitive to changes in the equitability of species, while the Shannon-Weaver index responds more to changes in the number of species present. (For more details concerning these indices, see



Magurran, 1988, Spellerberg, 1991 or Simpson 2002.)

While these indices can be useful indicators of the health of an area, a few cautions should be mentioned. Few surveys accurately tally all species due to lack of money, time, and available expertise in identifying all species. Important habitats are often inaccessible. There are also problems concerning the nature of the indices. Both of the indices discussed here treat all species as if they are interchangeable. A plot made up of native species with a moderate index value is considered less healthy than a second plot with a higher index but a greater number of exotic invaders. The indices also ignore the importance of keystone and indicator species. Finally, both the Simpson and Shannon indices assume that each species is equally different from all other species. In reality, this is not the case. For example, two species from the same genus are more alike than two species from different families. The Shannon index gives greater diversity value to a plot with many, equally abundant species that are very similar than it would to a plot with fewer, but less closely related, species. One way to measure the degree of difference between two species is through a distance function (see Weitzman 1992, 1993 and 1998; Solow et al. 1993 and Weikard 2002). The distance between two species may be measured in terms of the genetic differences between the two, the time since they branched off from some common ancestor or a weighted sum of differences in micro-characteristics. The greater the distance between the two species, the more different they are. To determine diversity of an area using distance functions, a series of pairwise distances between all species present must be calculated. With slight adjustments to the formulation of the distance function, species richness and abundance (such as is measured using the Simpson or Shannon index) can be incorporated. Because of the computational complexity of using distance functions, we use the Shannon Entropy index to measure biodiversity.

To examine how biodiversity varies over time, we must specify dynamics for the populations of species and the interaction between species. We model a conservation organization as choosing land parcels from  $H$  different available parcels. The conservation organization's objective is to maximize global diversity through parcel management. Due to a limited budget, not all land parcels may be preserved. To consider optimal parcels, the conservation organization must minimize the risk level for a given expected level of biodiversity.

In addition to distinct parcels, we also assume there are  $N$  possible species in nature (each species does not necessarily inhabit all parcels). The population of species  $i$  on parcel  $h$  at time  $t$  is denoted:  $y_{i,h,t}$ . Assuming the minimum population level is satisfied, we model the dynamics of the population of species as

$$y_{i,h,t} = y_{i,h,t-1} + \sum_{j=1}^J b_{i,j} \cdot \varepsilon_{j,h,t} + \sum_{k=1}^K g_{i,k} \cdot \nu_{k,t} + (1 - w_h) \cdot m_{i,h} \cdot \mu_t, \quad (1)$$

where  $b_{i,1}, \dots, b_{i,J}$  represent  $J$  parcel specific load factors that impact species  $i$ 's population,  $\varepsilon_{j,h,t}$  represents the  $j^{\text{th}}$  factor shock on parcel  $h$  at time  $t$ . Parcel-specific shocks may include natural disasters such as drought or forest fires.  $g_{i,1}, \dots, g_{i,K}$  represent  $K$  species global load factors for species  $i$ ,  $\nu_{k,t}$  represents the  $k^{\text{th}}$  global factor shock at time  $t$ . Global warming is the most well known potential global shock. Finally,  $m_{i,h}$  represents the humankind load factor for species  $i$  on parcel  $h$ ,  $w_h$  represents the proportion of parcel  $h$  preserved by the conservation organization, and  $\mu_t$  represents the global humankind shock at time  $t$ . We model the parcel specific shocks:

$$\varepsilon_{.,h} \equiv \begin{pmatrix} \varepsilon_{1,h} \\ \vdots \\ \varepsilon_{J,h} \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}, \Sigma_b \right), \text{ the global shocks } \nu \equiv \begin{pmatrix} \nu_1 \\ \vdots \\ \nu_K \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}, \Sigma_g \right) \text{ and the humankind shock}$$

$\mu \sim N(-1, \Sigma_m)$ . The parcel, global and humankind specific shocks are assumed to be independent (i.e.,  $\varepsilon_{.,h} \perp \nu$  for all  $h=1, \dots, H$ ) as are the  $H$  parcel specific shock vectors. Taking the expectations of (1) yields the expected population of species  $i$  on parcel  $h$  at time  $t$  or

$$E[y_{i,h,t}] = y_{i,h,t-1} - (1 - w_h) \cdot m_{i,h}, \quad (2)$$

with the variance given by

$$\text{Var}(y_{i,h,t}) = \sum_{j_1=1}^J \sum_{j_2=1}^J (b_{i,j_1} \cdot b_{i,j_2} \cdot \sigma_{j_1 j_2}) + \sum_{k_1=1}^K \sum_{k_2=1}^K (g_{i,k_1} \cdot g_{i,k_2} \cdot \theta_{k_1 k_2}) + (1 - w_h)^2 \cdot m_{i,h}^2 \cdot \phi^2, \quad (3)$$

where  $\sigma_{j_1 j_2}$  is the  $j_1$  row,  $j_2$  column element of  $\Sigma_b$  and  $\theta_{k_1 k_2}$  is the  $k_1$  row,  $k_2$  column element of  $\Sigma_g$ . Obviously, different species on the same parcel may interact in a variety of ways:

predator/prey, competition for similar food sources, symbiotic relationships. The interaction between species  $i_1$  and  $i_2$  on parcel  $h$  at time  $t$  is given by the covariance,

$$\text{Cov}(y_{i_1,h,t}, y_{i_2,h,t}) = \sum_{j_1=1}^J \sum_{j_2=1}^J (b_{i_1,j_1} \cdot b_{i_2,j_2} \cdot \sigma_{j_1 j_2}) + \sum_{k_1=1}^K \sum_{k_2=1}^K (g_{i_1,k_1} \cdot g_{i_2,k_2} \cdot \theta_{k_1 k_2}) + (1 - w_h)^2 \cdot m_{i_1,h} \cdot m_{i_2,h} \cdot \phi^2. \quad (4)$$

In addition to the interaction of different species on the same parcel, the same species interact on different parcels as individuals or groups migrate in order to mate or to expand their territory.

This interaction is given by the covariance between the same species on parcels  $h_1$  and  $h_2$  at time  $t$  or

$$\text{Cov}(y_{i,h_1,t}, y_{i,h_2,t}) = \sum_{k_1=1}^K \sum_{k_2=1}^K (g_{i,k_1} \cdot g_{i,k_2} \cdot \theta_{k_1 k_2}) + (1 - w_{h_1}) \cdot (1 - w_{h_2}) \cdot m_{i,h_1} \cdot m_{i,h_2} \cdot \phi^2. \quad (5)$$

Finally, different species interact across different parcels which is given by the covariance between species  $i_1$  and  $i_2$  on parcels  $h_1$  and  $h_2$  at time  $t$  or

$$Cov(y_{i_1, h_1, t}, y_{i_2, h_2, t}) = \sum_{k_1=1}^K \sum_{k_2=1}^K (g_{i_1, k_1} \cdot g_{i_2, k_2} \cdot \theta_{k_1 k_2}) + (1 - w_{h_1}) \cdot (1 - w_{h_2}) \cdot m_{i_1, h_1} \cdot m_{i_2, h_2} \cdot \phi^2. \quad (6)$$

Given this stochastic behavior of species on the available parcels, we will suppose the objective of our planner is to maximize global diversity. For this exercise, at least, we will take the measure of global diversity to be the Shannon index. If we define species  $i$ 's global

population at time  $t$  as:  $Y_{i,t} \equiv \sum_{h=1}^H y_{i,h,t}$  and the aggregate global population as:  $\Psi_t \equiv \sum_{i=1}^N Y_{i,t}$ , we

may formally state the planner's objective function to be maximized as:

$$S(y) = - \sum_{i=1}^N \left( \frac{Y_{i,t}}{\Psi_t} \cdot \ln \left[ \frac{Y_{i,t}}{\Psi_t} \right] \right). \quad (7)$$

The above index takes on its maximum when all species are equally represented in the parcels total population. By selecting parcels with the greatest index value, we are selecting parcels closest to this standard.

As noted, one disadvantage of the Shannon index is that it places equal weights on species (*i.e.*, a mosquito receives the same weight as a koala bear; a foundation species receives the same weight as a non-essential species). Perhaps a weakness in the Shannon index we have presented thus far is this implicit indifference between species. That is, interchanging the populations of mosquitoes with koalas does not impact the Shannon index value. If our conservation organization has a preference ordering of the species, the Shannon index will fail to convey this ordering. To allay these concerns, we may adapt our Shannon index to include a cross-entropy formulization.

In statistics, a cross-entropy formulization allows the analyst to impart prior information into the estimation process (Golan *et al*, 1996). In a biodiversity application, we can use the

cross-entropy specification to assign weights to the species. To incorporate the preferences of the conservation organization, higher weights may be assigned to more preferred species.

To allow for the assignment of greater benefits to a particular species, consider a *cross-entropy* index,  $C$ ,

$$C(y, q) = - \sum_{i=1}^N \left( \frac{1}{q_i} \frac{Y_{i,t}}{\Psi_t} \cdot \ln \left[ \frac{1}{q_i} \frac{Y_{i,t}}{\Psi_t} \right] \right) \quad (8)$$

where  $q$  is a  $N$  by 1 vector of species weights. The sum of the components of  $q$  must equal to 1 (each element is nonnegative).

Figures 1 and 2 illustrate the effects of incorporating preference weights on species' populations in the simple case with only two species. The curve in Figure 1 represents the value of the Shannon index with no preference weights. The horizontal axes measure the percentage of the parcels populations devoted to each species. As the figure indicates, the Shannon index attains the highest value when each species is equally represented in the parcel's population. Also, the curve is symmetric about this maximization point. This demonstrates the property of the Shannon index that allows us to interchange species populations 1 and 2 and not impact the index value.

This is not the case if we incorporate preference weights on the species. For example, Figure 2 was produced from the incorporation of preference weights on the species by using (8). In particular, species 2 was assigned a larger weight than species 1. The result, as Figure 2 demonstrates, is that the curve is no longer maximized at the point where the species' populations are equally represented on the parcel. The maximization point occurs where species 2 has the larger population. This reflects the preference weights we have assigned. Also, the curve is not symmetric. Hence, interchanging species populations will not result in analogous

biodiversity index values. This simple inclusion of the cross-entropy formulization may increase the efficacy of the Shannon index approach. However, it presents us with an additional dilemma. That is, how do we assign the species preference weights in practice?

The answer is that it likely depends on the preferences of the conservation organization. The preference weights may indicate economic value or expectation of contributions to medical technologies. The appealing notion of including a cross-entropy formulization is that preference orderings can be included in the analysis of biodiversity management. However, since the preference orderings of the conservation organization is unknown, we will use the unweighted Shannon index as shown in (7).

### **3. The Effects of Many Different, Spatially-Diverse Preserves or “Don’t Put All Your Genotypes in One Basket”**

To illustrate the effects of increasing the number of preserves on the overall total risk, measured in terms of variance of the Shannon index, we allow the number of equal sized parcels to increase. As we have 10 interrelated parcels to manage, we can begin illustrating the gain from managing biodiversity as a portfolio theory application.<sup>1</sup> Multiple parcels allow the manager to diversify away from the parcel specific shocks and use the covariance between parcels to select an optimal combination of portfolios given the global shocks. These global shocks represent the portion of risk to biodiversity that is common or systematic to all parcels, i.e., the risk that cannot be reduced by increasing the number of projects.

Figure 3 illustrates the impact of parcel holdings on portfolio risk. The vertical axis

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<sup>1</sup>Markowitz (1952) is considered to be the seminal portfolio theory work. For a modern overview, see Bodie et al (2008).

measures standard deviation of our portfolio of biodiversity index values. The horizontal axis indicates the number of parcels held in the portfolio. As the graph indicates, increasing the number of parcels reduces the risk to our portfolio biodiversity index value. If the manager selected only one parcel, his portfolio biodiversity value would fluctuate with the biodiversity of that single parcel. In this case, that would entail a standard deviation in biodiversity index value of approximately 0.14. If the manager equally allocated his resources between parcel 1 and parcel 2, the variance in his portfolio biodiversity index value would fall as Figure 3 indicates. As the number of parcels increase, the overall risk to biodiversity decreases to a point (see Figure 3). In general, we can demonstrate this result analytically. Consider a conservancy organization who acquires parcels 1 and 2. Its portfolio biodiversity is then:  $D = \frac{1}{2} \cdot S_1 + \frac{1}{2} \cdot S_2$ , where  $D$  is the weighted average of diversity,  $S_1$  and  $S_2$  represent the biodiversity index on parcels 1 and 2, respectively. The portfolio's variance in biodiversity is:  $Var(D) = \frac{1}{4}(\sigma_1^2 + \sigma_2^2) + \frac{\sigma_{12}}{2}$ . As the number of the parcels increase, the total variance approaches the average covariance. Simply noting that increasing the number of parcels preserved will reduce the risk to biodiversity does not answer the question of which specific parcels should be preserved. Also, the typical portfolio selection that maximizes expected utility places restrictions on the distribution of returns. Specifically, portfolio selection models that use a mean-variance criteria requires that the distribution of parcel returns be jointly normal.

#### **4. Stochastic Dominance as a Means of Project Selection**

To select optimal individual parcels, we use stochastic dominance. Although it has its

origins in financial economics, stochastic dominance has been employed in various applications in agricultural and environmental economics.<sup>2</sup> Stochastic dominance requires no distributional restrictions nor exact form of the utility function is required. To employ stochastic dominance, one needs only to make assumptions regarding the general characteristics of the conservation organization's utility function with greater restrictions arising with higher orders of stochastic dominance (see Levy 1992).

In the simplest form, a risky project,  $h_1$ , has stochastic dominance over risky project,  $h_2$ , if  $h_1$  delivers a preferred outcome relative to  $h_2$  in every state of nature. There are three forms of stochastic dominance, each with progressively stronger assumptions regarding a conservation organization's preferences. First-order stochastic dominance assumes only that conservation organizations prefer more to less or  $U(h_1) > U(h_2)$  where  $U(\bullet)$  is the conservation organization's utility function and parcel  $h_1$  is richer in biodiversity than parcel  $h_2$ . If this is the case, the stochastically dominant parcel is the one that provides the conservation organization with greater returns in every state of nature. More formally, parcel  $h_1$  is preferred to parcel  $h_2$  if, when returns are ranked from lowest to highest, the cumulative probability of returns for  $h_1$  is never greater and is sometimes less than the cumulative probability of returns for  $h_2$ .

Second-order stochastic dominance makes the additional assumption that conservation organizations are risk averse. If conservation organizations prefer more to less and are risk averse, then parcel  $h_1$  dominates parcel  $h_2$  with second-order stochastic dominance if the sum of the cumulative probabilities of all returns is never greater with  $h_1$  than with  $h_2$  and is sometimes

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<sup>2</sup>Other applications include conservation payments (Benítez et al. 2006), soil and water conservation (Bekele 2005) and community supported agriculture (Sanneh et al. 2001).



less. By making this additional assumption, we can select the optimal parcel if the two cumulative frequency functions cross. With second order stochastic dominance, it is no longer necessary that parcel  $h_1$  provides higher returns in every state of the world. Because of the assumption of risk aversion, the conservation organization is willing to accept the probability of lower returns in some middle range with parcel  $h_1$  in order to have a lower probability of a very low return.

Based on the earlier description of stochastic dominance, parcel  $h_1$  is said to second-order stochastically dominant parcel  $h_2$  if:

1. the planner prefers higher diversity to less,
2. the planner is risk-averse, and
3. the sum of cumulative probabilities for all diversity values are never more with  $h_1$  than  $h_2$  and sometimes less.

To explore the implications of this stochastic dominance as a parcel selection mechanism, we have created 10 parcels ( $H=10$ ). The number of distinct species is set to be 100 ( $N=100$ ). For the purposes of this simulation, we remove any influence of human development shocks. To do this, we set  $m_{i,k}$  to zero for all  $i$  and  $k$ . We allow for 12 parcel specific shock factors ( $J=12$ ) and 6 global shock factors ( $K=6$ ). The parcel specific loading factors are randomly assigned according to:  $b_{i,j} \sim Uniform(-1,1)$ . Similarly, the global loading factors are assigned according to:  $g_{i,k} \sim Uniform(-1,1)$ . The variance-covariance matrix  $\Sigma_b$  and  $\Sigma_g$  appear in Tables 1 and 2, respectively.

Once these parameterizations have been made, we have a stochastic distribution for

parcel diversity. The cdf of these distributions appear in Table 3. The first column of Table 3 lists values of the biodiversity measure,  $S$ , while the remaining 10 columns correspond to cdf values for each of the ten parcels. This table, for example, indicates that parcel 8 has a 1.2% chance of observing a biodiversity measure less than 3.06. Parcel 9 has a 0.2% chance of observing a biodiversity measure less than 3.06.

Our task is now to rank these parcels in accordance with stochastic dominance. To demonstrate the parcel selection problem under uncertainty with the measure of biodiversity presented above, we propose the following simulation. We have 10 ( $H=10$ ) parcels available that our decision maker must choose from in conserving. There are 100 ( $N=100$ ) distinct species that may inhabit each parcel. Each species was sequentially populated on each of the 10 parcels. The algorithm used to define the initial population was  $y_{i,h,1} = 0.45i(6 - h) + 10$  for parcels 1 through 5. For parcels 6 through 10, the initial population was defined as

$y_{i,h,1} = 0.45(100 - i)(h - 5) + 10$ . This structure creates a gradation of population characteristics across the ten parcels with some intended symmetries. Parcel 1 and 10 are closely related. For example, species 1 on parcel 1 has the same initial population as species 100 on parcel 10. The parcel pairs (2, 9), (3, 8), (4,7), and (5, 6) share the same relationship.

This does not imply that the stochastic behaviors of the parcel specific biodiversity measures are analogous. This is because we are not imposing the shock loading factors to be equivalent. The shock impact factors (parcel specific and global) were uniformly selected between -1 and 1. The variance-covariance structure of the parcel specific shock terms was taken from an empirical variance-covariance structure of 12 ( $J=12$ ) financial factors. The variance-covariance structure of the global shock terms was taken from an empirical variance-

covariance structure of 6 ( $K=6$ ) parameters in a demand study. These choices were selected out of convenience as they were readily available.<sup>3</sup> Our purpose in this simulation is to demonstrate the impact of uncertainty on our biodiversity and parcel selection modeling. The heterogeneity in impact shock assignments will generate different biodiversity measure experiences across the distinct parcels. Also, before the experiment, we have no a priori reason to find one parcel as superior to the other. While some parcels have high overall populations (particularly parcels 1 and 10), others have greater overall initial biodiversity (especially parcels 5 and 6). We intentionally impose these conflicts to mimic the tradeoffs conservation managers confront in making decisions.

Given the initial population settings, the biodiversity measures are initially highest for parcels 5 and 6 even though these parcels have the smallest initial population. The parcel population is more uniformly assigned across species for each of parcels 5 and 6. On parcels 5 and 6, the most populous species is approximately 5.26 times larger than the least populous species. On parcels 1 and 10, the most populous species is about 19.18 times larger than the least populous species. While parcels 5 and 6 have highest initial species biodiversity, their smaller species populations are also closer to the minimum sustainable values. This suggests the parcels are likely to lose more species to extinction given particular negative species shocks.

With these model parameters defined, we may now explore the ramifications of uncertainty on our biodiversity index. For each simulation period, we drew the 12 parcel specific shock terms for each of the 10 parcels (120 parcel specific shock terms) and the 6 global shock terms. Thus, there are a total of 126 random draws each period. We then updated the

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<sup>3</sup>The correlation matrix for the specific shocks is reported in Table 1 while the variance-covariance structure of the global shock terms is given in Table 2.

species carrying-capacities on each parcel. We did this for 500 periods for a complete simulation. At the end of the 500 periods, we calculate each parcel's biodiversity measure (8,000 random components as well as the defined parameters influence each parcel's ending biodiversity measure).

One simulation, however, only produces a single possible outcome, not necessarily the most likely, of biodiversity. That is, a single observation of a random variable. To investigate the stochastic behavior of biodiversity on our parcels, we constructed simulated cumulative density functions (cdf) for each parcel. Each parcel had 1,000 replications of 500 periods. The cdfs are shown for a few parcels in Figure 4. The vertical axis measures the probability of observing a biodiversity index equal to or less than the horizontal distance to the curve. The horizontal axis denotes the biodiversity index value for the parcel. The parcels are ordered by first order stochastic dominance. The curve furthest to the left reflects a stochastically dominated parcel. That is, all other parcels stochastically dominate it as the probability of observing a particular biodiversity index value is always less for the remaining parcels.

As we inspect the parcels further to the right in Figure 4, we see that a strict ordering of the parcels in accordance with first-order stochastic dominance is not possible. This is because their cdfs intersect one another. This indicates that in some ranges we have a parcel stochastically dominated by another, but the order of stochastic dominance reverses in this range after the cdfs cross. This implies first-order stochastic dominance fails to identify a ranking for all parcels exhibiting this behavior.

Figure 5 illustrates that examining the parcels with respect to second order stochastic dominance reduces this problem. Recall that second order stochastic dominance adds the reasonable assumption that conservation organizations are risk averse. By adding this

assumption and reporting the sum of the cdfs, Figure 5 allows us to order parcels that we could not previously rank. We can now clearly identify which of these would be preferred. However, there remain three parcels whose curves still cross, which are not included in this figure. While the use of second order stochastic dominance is therefore an improvement in ranking parcels, it is not able to do so in every case. However, all parcels can be ranked with 4<sup>th</sup> order level stochastic dominance.

## **5. Policy Implications and Conclusions**

Diversification in the world of finance refers to holding a variety of assets in order to reduce risk. In the world of biology, diversification in the form of preserving similar habitats in geographically diverse areas serves the same purpose. The risks to biodiversity come from a variety of sources, some associated with human actions, some not. Because some of these risks affect certain areas more severely than others, organizations concerned with protecting biodiversity can benefit from applying financial strategies for diversification. Using simulations in which both global and localized shocks are imposed on parcels that may be persevered, we show that increasing the number of persevered parcels reduces risk. However, like the risk associated with financial markets, not all risk to biodiversity (specifically, global shocks) can be eliminated through diversification.

We also consider the possibility of using stochastic dominance to rank parcels for preservation. When choosing specific areas to preserve, conservation organizations take into account both the returns to preservation, i.e. the level of biodiversity, and the risks to that biodiversity that arise due to shocks. Simulations indicate that stochastic dominance may be a useful tool in some cases, clearly indicating which parcels are preferred, but cannot differentiate

between all possible parcels.

The simulations included in this paper measure returns by using the Shannon-Weaver index of biodiversity. However, this index assigns equal weights to all species in an area. In reality, conservation organizations have indicated that they have definite preferences for some species over others. For example, they may prefer native species to exotic invasive species, or they may have greater concern for a species that acts as a foundation species. These preferences can be taken into account by using a cross-entropy formulization. Additional simulations using such a weighted index will provide further insight into decision making by conservation organizations.

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Figure 1  
Shannon Index and Population Allocation

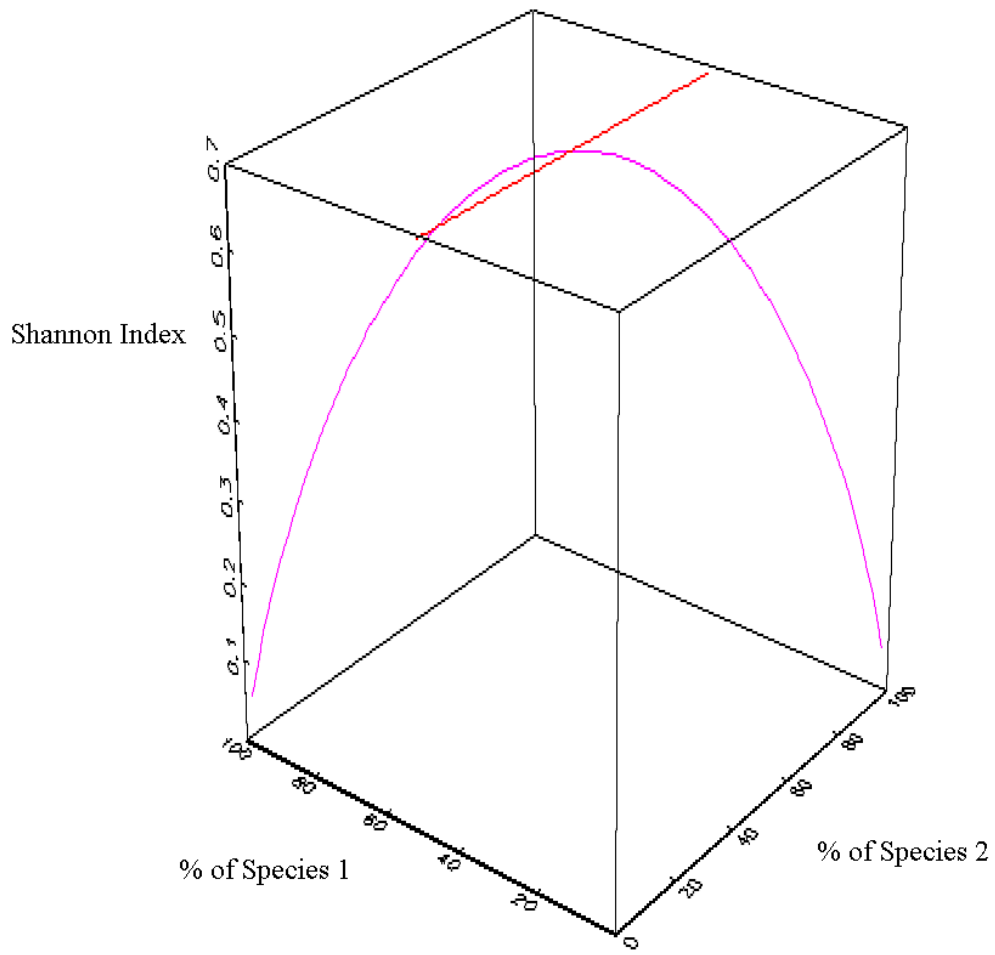


Figure 2  
Cross Entropy Index and Population Allocation

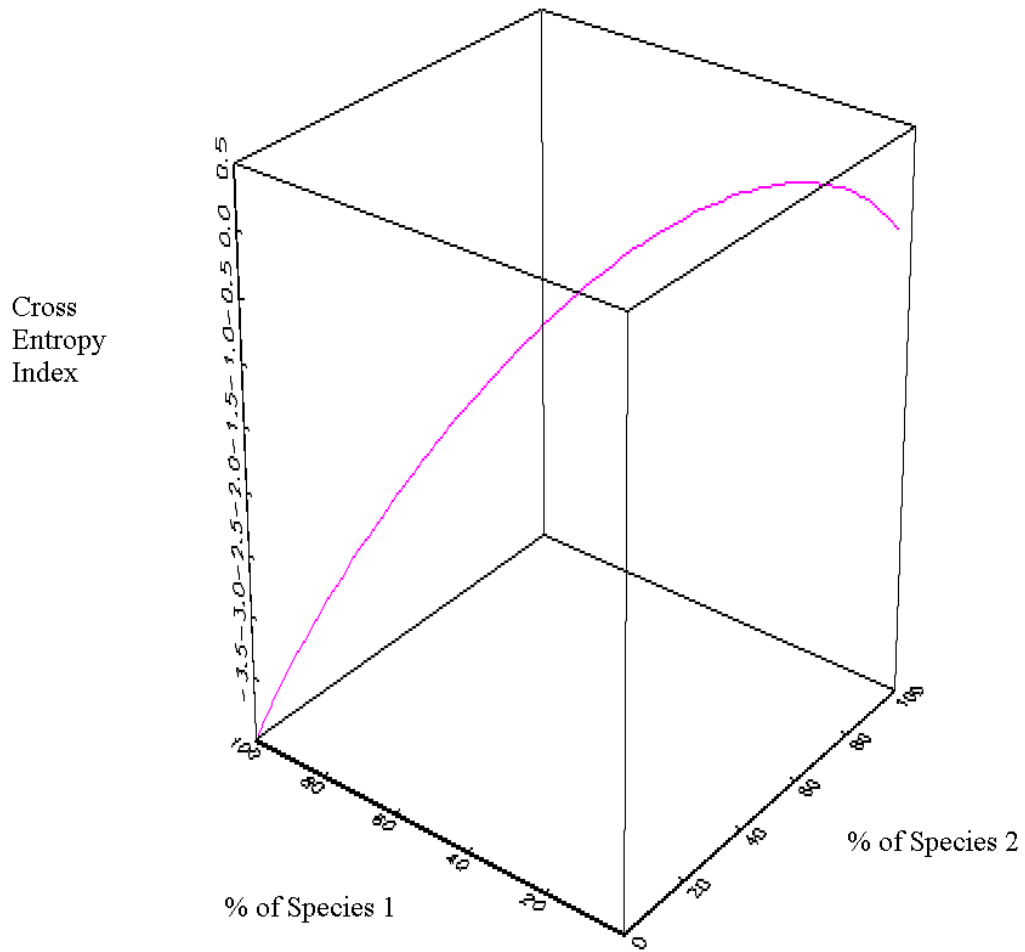


Figure 3  
Conservation Organization's Risk over Parcels

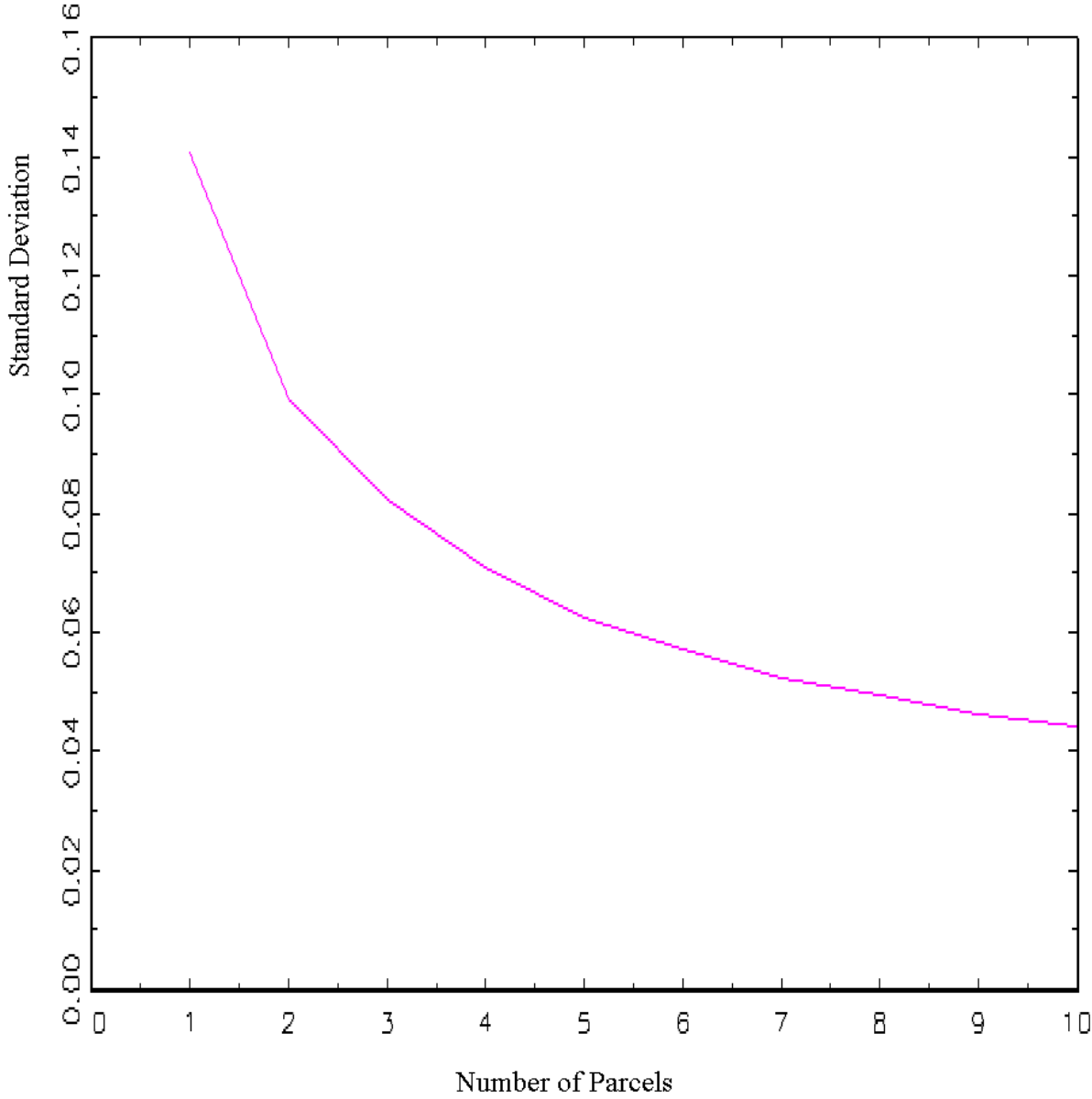


Figure 4  
Ordered by First Stochastic Dominance

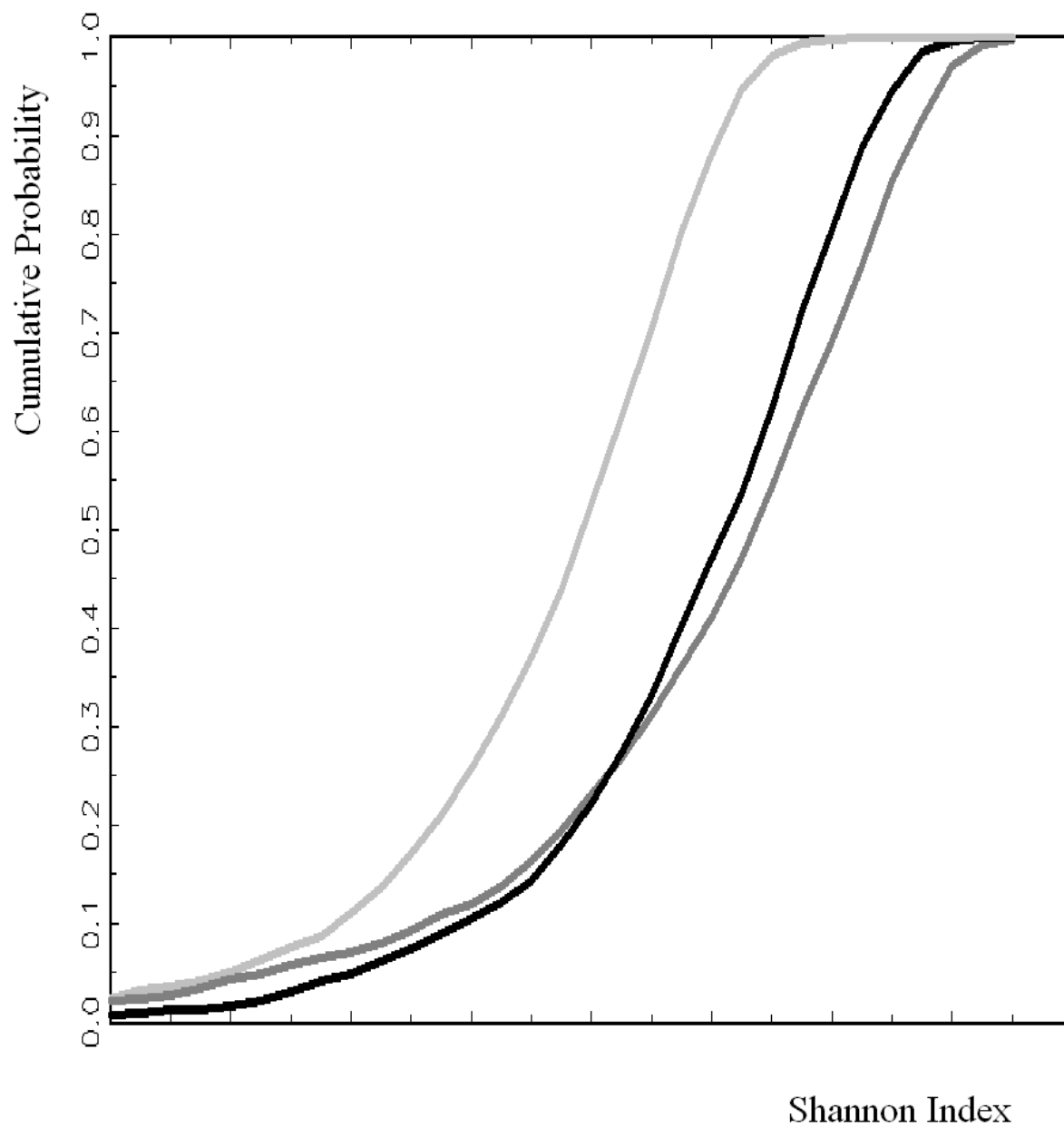
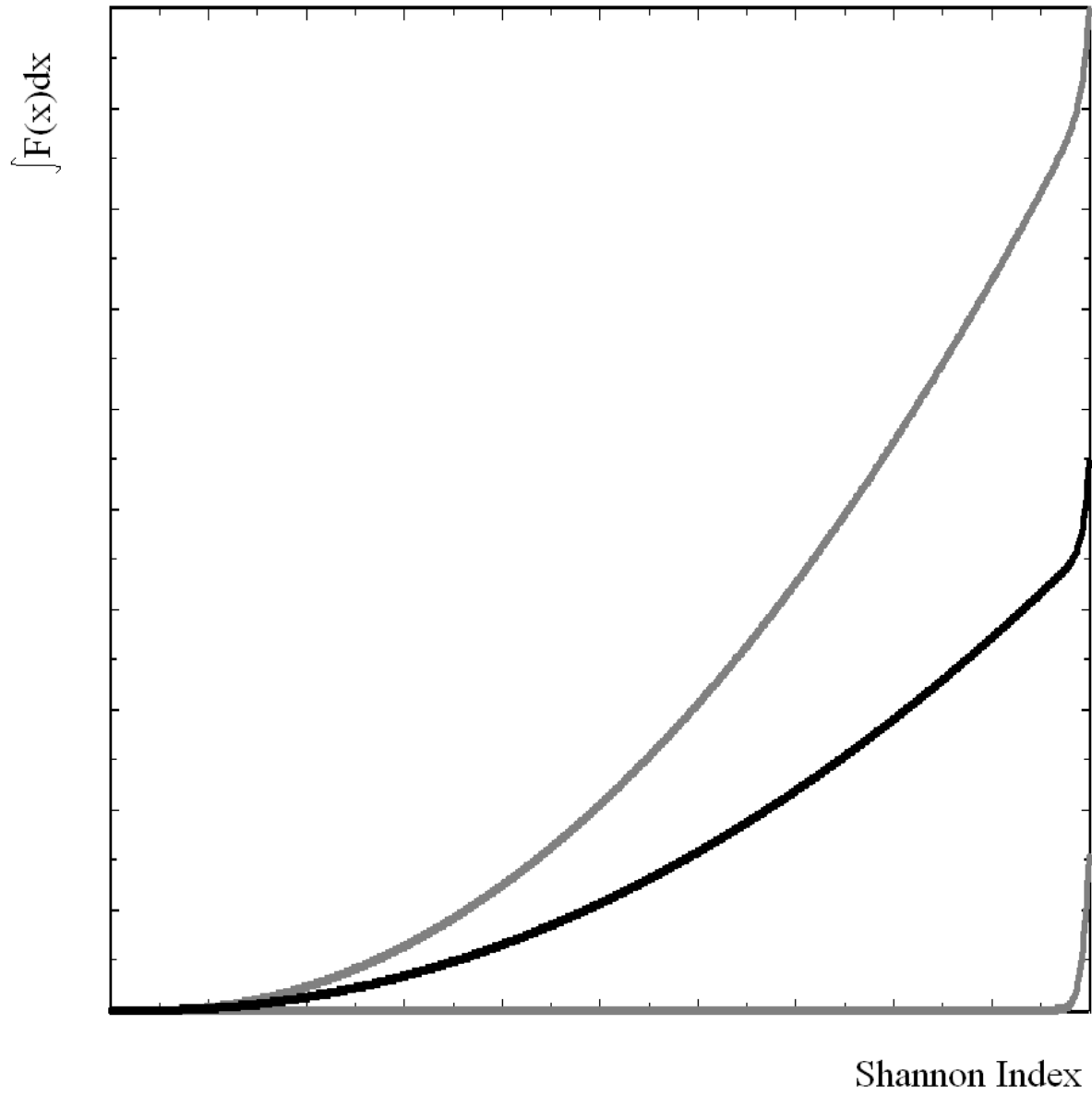


Figure 5  
Ordered by Second Stochastic Dominance



**Table 1: Parcel Specific Shock Correlations**

	$\varepsilon_1$	$\varepsilon_2$	$\varepsilon_3$	$\varepsilon_4$	$\varepsilon_5$	$\varepsilon_6$	$\varepsilon_7$	$\varepsilon_8$	$\varepsilon_9$	$\varepsilon_{10}$	$\varepsilon_{11}$	$\varepsilon_{12}$
$\varepsilon_1$	1	0.1	-0.5	0.5	0.6	-0.2	-0.1	0.4	0.2	0.0	0.3	-0.5
$\varepsilon_2$	0.1	1	0.2	-0.0	0.3	-0.2	-0.6	-0.3	-0.1	0.1	0.1	-0.3
$\varepsilon_3$	-0.5	0.2	1	-0.2	-0.3	0.1	-0.1	-0.2	0.0	0.2	-0.2	0.2
$\varepsilon_4$	0.5	-0.0	-0.2	1	0.4	-0.0	0.0	0.4	0.1	0.1	0.1	-0.3
$\varepsilon_5$	0.6	0.3	-0.3	0.4	1	-0.4	-0.4	0.3	0.2	0.1	0.5	-0.8
$\varepsilon_6$	-0.2	-0.2	0.1	-0.0	-0.4	1	0.4	-0.2	-0.1	-0.0	-0.1	0.3
$\varepsilon_7$	-0.1	-0.6	-0.1	0.0	-0.4	0.4	1	0.2	-0.2	-0.2	-0.3	0.5
$\varepsilon_8$	0.4	-0.3	-0.2	0.4	0.3	-0.2	0.2	1	0.2	-0.1	-0.1	-0.0
$\varepsilon_9$	0.2	-0.1	0.0	0.1	0.2	-0.1	-0.2	0.2	1	0.2	0.2	-0.2
$\varepsilon_{10}$	0.0	0.1	0.2	0.1	0.1	-0.0	-0.2	-0.1	0.2	1	0.3	-0.2
$\varepsilon_{11}$	0.3	0.1	-0.2	0.1	0.5	-0.1	-0.3	-0.1	0.2	0.3	1	-0.5
$\varepsilon_{12}$	-0.5	-0.3	0.2	-0.3	-0.8	0.3	0.5	-0.0	-0.2	-0.2	-0.5	1

**Table 2:** Global Shocks Variance-Covariance Structure

	$v_1$	$v_2$	$v_3$	$v_4$	$v_5$	$v_6$
$v_1$	0.132	0.004	0.002	0.002	0.000	-0.004
$v_2$	0.004	4.034	2.002	0.361	0.073	-3.207
$v_3$	0.002	2.002	1.135	0.159	0.041	-1.646
$v_4$	0.002	0.361	0.159	1.395	0.0483	-0.459
$v_5$	0.000	0.073	0.041	0.0483	0.003	-0.065
$v_6$	-0.004	-3.207	-1.646	-0.459	-0.065	2.594

**Table 3: Biodiversity cdf**

S	1	2	3	4	5	6	7	8	9	10
3.06	.006	.002	.002	.001	.008	.003	.004	.012	.002	.004
3.15	.015	.012	.003	.001	.034	.003	.008	.034	.005	.020
3.23	.038	.030	.012	.004	.103	.013	.028	.093	.019	.065
3.32	.097	.075	.026	.017	.241	.035	.077	.212	.059	.184
3.41	.220	.169	.066	.054	.449	.106	.175	.397	.156	.393
3.49	.437	.329	.168	.156	.693	.231	.389	.641	.330	.626
3.58	.665	.570	.344	.308	.889	.446	.657	.848	.548	.823
3.66	.873	.799	.562	.543	.979	.672	.853	.964	.781	.952
3.75	.973	.939	.764	.775	.995	.864	.957	.996	.915	.991
3.83	.997	.987	.913	.923	.999	.970	.995	1.000	.983	.999