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Substitution and Superstars

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by

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Abstract

The existing superstar model (Rosen 1981) does not require imperfect substitutes and explains the convexity of *total earnings* with respect to talent due to higher output for those with the most talent. We develop a model that explains why *per unit earnings* (wages or prices) would increase at an increasing rate in talent. Imperfect substitution results due to the probabilistic nature of production. Costs to consumers from repeated consumption---multiple surgeries for example---are neither necessary nor sufficient for convexity in wages.

JEL classifications: D11, D31, and J31

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1. Introduction

It has been thirty years since the publication of Sherwin Rosen's seminal paper on superstars (Rosen, 1981). In that paper, and in a non-technical paper of the same title published two years later (Rosen, 1983), Rosen analyzed markets that he argued contain at least one of two features: poor talent is an inadequate substitute for superior talent (superstars), and technology is such that many buyers can be served simultaneously (there is low marginal cost of providing additional units of the service), as with *joint goods*. These features may lead to total earnings for an individual or firm increasing at an increasing rate in talent (convexity in earnings), and a few high quality individuals or firms selling a large percentage of market output and reaping a large percentage of market revenue.

Rosen (1981) noted that imperfect substitutability can not account for a significant concentration of output among a few sellers, nor is imperfect substitutability necessary to explain convexity in total earnings. He also briefly considered the case developed in more detail in Perri (2011) in which perfect substitutes are assumed. In that case, convexity of earnings results because higher quality sellers have both higher prices and output. Marginal cost increases in output, and may even increase in quality, provided it does not increase too rapidly in the latter. Superstars may earn a disproportionate share of revenue, but do not produce a significant percentage of market output. In order to have large percentages of market output accruing to a few sellers, low marginal cost is required.

Thus, imperfect substitutability is not required for either convexity of total earnings (total revenue for a firm) or a large concentration of market output and earnings among a few sellers. Additionally, convexity in *per unit earnings* (wages or prices) can not be explained by low marginal cost of production. The purpose herein is to demonstrate how imperfect substitutability can explain convexity in the wage rate. We do so in a model of probabilistic success, where more than one non-superstar can have almost the same likelihood of accomplishing a task as a superstar. Thus, we offer an explanation of one superstar phenomenon not explained by Rosen's classic model. Also, we explicitly derive a measure of substitutability which is not present in the Rosen model.

Rosen mentions doctors as an example of superstars (Rosen, 1981). Further Rosen suggests a surgeon who is 10% more likely to save a life should earn much more than a 10% premium. He also mentions lawyers as an example of poor substitutability of lesser talent for superior talent (Rosen, 1983). A low marginal cost of serving many customers does not characterize the market for lawyers and doctors. Nor does it explain why the real earnings of the highest paid dentists tripled from 1979 to 1989 while average dental earnings barely increased (Frank and Cook, 1995). Convex earnings profiles for surgeons, lawyers, and dentists must be due to the *wage rate* for these individuals increasing rapidly in talent. This contrasts with *media markets* (Borghans and Groot, 1998) such as television, movies, and recorded music, where a few individuals may capture much of a market in terms of output and revenue. There low marginal cost certainly exists.

The outline of the rest of the paper is as follows. In Section 2, we discuss imperfect substitution. In Section 3, we develop the formal model of imperfect substitution for an individual consumer. Since hiring more than one non-superstar may imply a sequence of hires, there could be costs to consumers due to delay in accomplishing the desired task. Such costs are considered in Section 4. It is shown such costs are neither necessary nor sufficient for convexity in the wage rate. Market equilibrium is considered in Section 5, and Section 6 contains a summary and discussion of possible future work.

2. Substitution

The usual view of a superstar is one where "...lesser talent often is a poor substitute for greater talent.[1] Superstars occur when consumers place "...considerable weight on quality versus quantity."[2] In order to consider imperfect substitutes, we follow Rosen (1981) in two ways. First, Rosen's suggestion a 10% more successful surgeon should command a wage premium of more than 10% implies a probabilistic dimension to production, which provides a simple way to model substitutability. Second, Rosen argued:

> "Though sellers of different quality are imperfectly substitutable with each other, the extent of substitution decreases with distance. In the limit *very close neighbors are virtually perfect substitutes*" (emphasis added).[3]

We combine both of these points in order to consider the extent of substitutability between individuals, where more talented individuals are more likely to succeed in producing the service desired by consumers.

Consider employing one superstar who has a higher chance of success than that of non-superstars. What is of interest is the probability of success when more than one non-superstars are employed, with the possibility of both simultaneous and sequential use of non-superstars.

Rosen (1983) mentions, without elaborating, the case of two lawyers, each of whom individually has a 50% chance of winning a case. He suggests employing both lawyers might not elevate the probability of winning much above 50%, and might actually decrease the likelihood of winning. However, unless one lawyer impedes another, the probability of winning should increase as more lawyers are employed. Clients often engage teams of lawyers. Presumably more

[1] Rosen, 1981, p.846.
[2] Autor, 2005, p.2.
[3] Rosen, 1981, p.850.

non-superstars (possibly working fewer hours) could substitute for some number of superstar lawyers. Surgeons also work in teams, and it is possible to have more than one surgeon present even if only one actually performs the surgery.

One way of considering substitution is to suppose talent means the likelihood one will correctly determine how to proceed with a task---a case for lawyers, an operation for a surgeon, etc. In Rosen's example, one can think of employing two lawyers, *A* and *B*, each of whom has a 50% of succeeding. If one lawyer can not determine how to proceed, the view of the other is immediately considered (that is, there is no delay of any consequence). The probability the two lawyers are successful is then

{probability of success for *A*} + {probability of failure for *A*}{probability of success for *B*} =

$5 + .5^2 = .75$.

The same probability of success might be attainable if the lawyers were hired sequentially: first one trial occurs, and then, if one is not successful, either a second similar trial (if the first resulted in a hung jury) or an appeal (if the first trial is lost) occurs. For now, we ignore delay costs (possibly lost work time for a second trial, and incarceration awaiting a second trial if convicted in the first trial). It is possible the two trials may not be independent events: losing the first trial might affect the likelihood of winning the second, given lawyer talent. However, assuming independence allows a simple way to consider imperfect substitutes, and may be a reasonable approximation of reality.

Using the idea from Rosen that very close services are essentially perfect substitutes, suppose n non-superstars can produce a likelihood of success equal to λ times the probability a superstar would succeed, with λ possibly very close to one. If an individual would pay v for the services of a superstar an individual would be willing to pay λv for the *combined services of the* *n superstars*. Further, to emphasize Rosen's point substitutability diminishes with distance, it is assumed a success rate less than λ has no value to the consumer. Individuals decide what value of λ is acceptable to them. In the next section, it will be shown a larger value of λ is naturally interpreted as less substitutability of non-superstars for superstars.

Rosen's (1981) method of explicitly deriving imperfect substitution involved a fixed cost of consumption per unit of quality. However, he recognized fixed consumption costs were not necessary for his results. As discussed previously, even perfect substitutes can result in earnings convex in talent and large shares of market output and earnings accruing to a few superstar sellers. We also will consider a cost of consumption which results from delay due to hiring a sequence of non-superstars. However, as noted above, such a cost is neither necessary nor sufficient for convexity of the wage rate with respect to talent. Convexity results from the probabilistic nature of production of services.

3. A model with substitution

Let λ be the probability of success when n non-superstars are employed (either simultaneously or sequentially). For simplicity, it is assumed the probability of success for a superstar is one. Each non-superstar has a success probability of p, $0 < p < 1$, and the success of one is independent of the success of others. Thus, employing n non-superstars, the probability of success is given by:

$$\lambda = p + (1-p)p + (1-p)^2p + ... + (1-p)^{n-1}p = 1 - (1-p)^n \qquad (1)$$

Solving *eq.*(1) for n:

$$n = \frac{ln(1-\lambda)}{ln(1-p)} \quad (2)$$

Note $|ln(1-\lambda)| > |ln(1-p)|$ for $\lambda > p$. The minimum value for λ is p, in which case $n = 1$. Suppose a superstar's value to a consumer is v. If $\lambda = p$, we have perfect substitutes: a consumer is indifferent to hiring one superstar at a wage of v or one non-superstar at a wage of pv. As λ increases above p, superstars and non-superstars become worse substitutes. With $\lambda > p$, $n > 1$.

Note, if an individual had a value of λ, call it $\hat{\lambda}$, lower than the value of p for some sellers, then, for $\hat{\lambda} \leq p$, these sellers are perfect substitutes for superstars, and will be paid pv. This possibility is not important for the analysis for an individual buyer, but will be considered again in Section 5 when the market for these services is considered.

As suggested by Rosen (1981), the extent of substitution decreases as the outcome with non-superstars is farther from that of superstars. Herein, it is assumed the decrease in substitutability is extreme. Consumers choose λ, which may be close to one. Thus, consumers decide at what level of success for a set of non-superstars, relative to that for superstars, the combined effort for the non-superstars is a perfect substitute for that of a superstar. If n non-superstars yield an expected probability of success of λ, a consumer would pay λv *in total* for the n superstars. However, given the desired level of λ,[4] it is assumed combinations of non-superstars that lead to a probability of success less than λ are not substitutes for superstars, and would not be hired. Again, given λ, consumers would only purchase the services of either a superstar, or of n non-superstars, with n determined by *eq*.(2).

To see the impact of λ and p on n, differentiate n:

[4] Differences in λ among consumers are considered in Section 5.

$$\frac{\partial n}{\partial p} = \frac{\ln(1-\lambda)}{(1-p)[ln(1-p)]^2} < 0, \tag{3}$$

$$\frac{\partial n}{\partial \lambda} = \frac{-1}{(1-\lambda)ln(1-p)} > 0. \tag{4}$$

As one would expect, as p increases or λ decreases, fewer non-superstars are required to replace a superstar. Consumers decide how close to the success of a superstar they require in order to treat the effort by n individuals as being equivalent to λv. Let $v = 1$. Then each non-superstar is valued by w:

$$w = \lambda/n = \frac{\lambda ln(1-p)}{ln(1-\lambda)}. \tag{5}$$

An increase in λ directly increases w because the probability of success of the non-superstars is higher. However, a larger λ requires a larger n, given p, which lowers w. We have:

$$\frac{\partial w}{\partial \lambda} = \frac{ln(1-p)}{[ln(1-\lambda)]^2}\left[ln(1-\lambda) + \frac{\lambda}{1-\lambda}\right]. \tag{6}$$

Now $\frac{\partial w}{\partial \lambda} < 0$ if $|ln(1- \lambda)| < \frac{\lambda}{1-\lambda}$. When $\lambda = 0$, $-ln(1-\lambda) = \frac{\lambda}{1-\lambda} = 0$. Also, $\frac{\partial\left(\frac{\lambda}{1-\lambda}\right)}{\partial \lambda} = \frac{1}{(1-\lambda)^2} > \frac{\partial[-ln(1-\lambda)]}{\partial \lambda} = \frac{1}{1-\lambda}$. Thus, $\frac{\lambda}{1-\lambda} > -ln(1-\lambda)$ for all $\lambda > 0$, and $\frac{\partial w}{\partial \lambda} < 0$. When non-superstars become worse substitutes for superstars, that is, when λ increases, the wage of non-superstars declines.

To see if the wage increases at an increasing rate in ability, p, differentiate w:

$$\frac{\partial w}{\partial p} = \frac{-\lambda}{(1-p)ln(1-\lambda)} > 0, \tag{7}$$

$$\frac{\partial^2 w}{\partial p^2} = \frac{-\lambda}{(1-p)^2 ln(1-\lambda)} > 0. \tag{8}$$

Thus, we have the wage convex in ability due to imperfect substitution. Since the wage represents per unit compensation, nothing herein depends on those with more ability (a larger *p*) selling more units of their service. Nor does convexity require the delay costs mentioned above and considered in the next section.

Further, we can see how λ affects the slope and convexity of the wage as a function of *p*:

$$\frac{\partial^2 w}{\partial p \partial \lambda} = \frac{-\left[ln(1-\lambda)+\frac{\lambda}{1-\lambda}\right]}{(1-p)[\ln(1-\lambda)]^2} < 0, \tag{9}$$

$$\frac{\partial^3 w}{\partial p^2 \partial \lambda} = \frac{-\left[ln(1-\lambda)+\frac{\lambda}{1-\lambda}\right]}{(1-p)^2[\ln(1-\lambda)]^2} < 0. \tag{10}$$

Although we must have $\lambda > p$ for there to be imperfect substitution, if superstars and non-superstars are worse substitutes ($d\lambda > 0$), the slope and convexity of *w* with respect to *p* decline. This is because $\frac{\partial w}{\partial \lambda}$ is negative.

Table One shows values for *n* and *w* for cases when superstars and non-superstars are poor substitutes (λ is close to 1). To illustrate the convexity of earnings (*w*) in talent (*p*), consider comparable percentage increases in *p*. For example, if $\lambda = .95$, an increase in *p* from .5 to .7 (a 40% increase in *p*) causes *w* to increase by 73% (from .22 to .38). A further increase in *p* from .7 to 1 (a 43% increase in *p*) results an increase in *w* of 163% (from .38 to 1). For an illustration of

Rosen's (1983) point that a surgeon who is 10% more successful in saving lives should be paid a good deal more a than 10% premium, compare superstars and those with p = .9. A superstar's success rate is about 11% higher than that of one with p = .9, but, if λ = .99, a superstar would be paid twice that of one with p = .9. With λ = .95, a superstar would still earn about 37% more than the a non-superstar who has p = .9.

4. Delay cost

As discussed in section one, if $n > 1$ implies a sequence of hiring of non-stars, there may be costs of delay. These are costs in addition to the total amount paid in wages, wn. Denote such costs as $c(n)$. Delay may be important for some activities and trivial for others. For example, if one hires a non-superstar lawn service to kill weeds, several visits by the service for a few weeks simply costs one a little longer time with an unsightly lawn. Alternatively, if one must have surgery, and n means more than one operation (versus one performed by a superstar) and not n surgeons performing one operation, delay cost would at least imply additional pain and suffering, and could be substantial.

Delay cost causes the wage for non-superstars to be even lower than what results due to imperfect substitution ($\lambda > p$). Let $\delta = c(n)/n$. Thus, δ is the average cost of delay (per non-superstar hired). With primes denoting partial derivatives, we have:

$$\frac{\partial \delta}{\partial n} = \frac{nc' - c}{n^2} = \frac{c}{n^2}\left(\xi_{c,n} - 1\right), \tag{11}$$

Table One. The wage (w) and number (n) of non-superstars (with the wage of superstars = 1).

λ	p	n	w
.999	.9	3	.333
.999	.8	4.29	.233
.999	.7	5.74	.174
.999	.6	7.54	.132
.999	.5	9.97	.1
.99	.9	2	.5
.99	.8	2.86	.355
.99	.7	3.82	.26
.99	.6	5.03	.2
.99	.5	6.64	.15
.95	.9	1.3	.73
.95	.8	1.86	.51
.95	.7	2.49	.38
.95	.6	3.27	.29
.95	.5	4.32	.22

where $\xi_{c,n}$ is the elasticity of *c* with respect to *n*. Thus, the average cost of delay is a positive function of *n* if delay cost is elastic in *n*. Note if $c = k(n-1)^{\theta}$, $k \geq 0$ & $\theta > 0$, then $\frac{\partial \delta}{\partial n} = \frac{k(n-1)^{\theta-1}}{n^2}[n\theta - (n-1)]$. Thus, $\frac{\partial \delta}{\partial n} > 0$ if $\theta > \frac{n-1}{n}$. If $\theta = 1$, so $c" = 0$, $\xi_{c,n} > 1$ and $\frac{\partial \delta}{\partial n} > 0$. Even if $c" < 0$, we can still have $\xi_{c,n} > 1$ if $\theta > \frac{n-1}{n}$.

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With delay cost, $w = \frac{\lambda ln(1-p)}{ln(1-\lambda)} - \delta$. We then have:

$$\frac{\partial w}{\partial p} = \frac{-\lambda}{(1-p)ln(1-\lambda)} - \frac{\partial \delta}{\partial n}\frac{\partial n}{\partial p}. \tag{12}$$

With the first term on the RHS of *eq*.(12) positive and $\frac{\partial n}{\partial p} < 0$, delay cost increases $\frac{\partial w}{\partial p}$ if $\frac{\partial \delta}{\partial n} > 0$---that is, if $\xi_{c,n} > 1$. To see how delay cost affects the convexity of *w*, the second derivative of *w* with respect to *p* is now:

$$\frac{\partial^2 w}{\partial p^2} = \frac{-\lambda}{(1-p)^2 ln(1-\lambda)} - \left\{\frac{\partial^2 \delta}{\partial n^2}\left[\frac{\partial n}{\partial p}\right]^2 + \frac{\partial \delta}{\partial n}\frac{\partial^2 n}{\partial p^2}\right\}. \tag{13}$$

With the first term on the RHS of *eq*.(13) positive, let the {•} term in *eq*.(13) be noted by *J*. If $J < 0$, delay cost makes *w* more convex in *p*.

$$J = \left[\frac{ln(1-\lambda)}{(1-p)[ln(1-p)]^2}\right]^2 [n^2(c' + nc" - c') - 2n(nc' - c)]\frac{1}{n^4}$$

$$+ \frac{(nc'-c)}{n^2(1-p)^2[ln(1-p)]^3} ln(1-\lambda)[ln(1-p) + 2]. \tag{14}$$

If $c'' < 0$, J is more likely to be negative, but, if $c'' > 0$, the term in J involving c'' is positive. If $c'' \approx 0$, we have:

$$J = \frac{cln(1-\lambda)\left(\xi_{c,n}-1\right)}{n^2(1-p)^2[ln(1-p)]^3}\left[ln(1-p) + 2 - \frac{2ln(1-\lambda)}{nln(1-p)}\right]. \qquad (15)$$

Using $n = \frac{ln(1-\lambda)}{ln(1-p)}$, *eq*.(15) becomes

$$J = \frac{cln(1-\lambda)\left(\xi_{c,n}-1\right)}{n^2(1-p)^2[ln(1-p)]^2}, \qquad (15a)$$

which is < 0 if $\xi_{c,n} > 1$. Thus, cost elastic in n and $c'' \leq 0$ ensure delay cost increases the magnitude of $\frac{\partial w}{\partial p}$ and the convexity of w in p. Although delay cost lowers the wage, given p, such cost is not necessary for w to be convex in p, nor does delay cost unambiguously increase either the first or second derivative of w with respect to p. Imperfect substitution is all that is necessary for the wage to increase at an increasing rate in ability (p).

5. Market equilibrium

Our previous analysis considered individual demand for non-superstars. We now consider labor market equilibrium when there are differences in λ among individuals. If there is a continuous distribution of consumers with respect to λ, we have the demand for any p.[5] For

[5] If w follows *eq*.(5), individuals are indifferent to what ability individuals are hired. The sum of the demand schedules for each level of p equals the market demand.

$\lambda \geq p$, those with the smallest value of λ---equal to p---value p the most. Since $\frac{\partial w}{\partial \lambda} < 0$, as λ increases, the wage one would pay for any p declines. Thus the distribution of consumers in the population with respect to λ traces out the demand for any p.

Consider the demand for two types of p, p_1 and p_2, with $p_1 < p_2$. As discussed in Section 3, for a buyer with a particular value of λ, sellers with $\lambda \leq p$ are perfect substitutes. Using Figure One, with $p_1 < p_2$, there will be more sellers who are perfect substitutes the higher the value for p. Thus point **a** lies to the right of point **b** in Figure One. Because of this, moving to the right of point **a**---where p_1 is no longer a perfect substitute for buyers who have $\lambda > p_1$---the vertical distance between the demands for the two ability types is larger than would be the case if point **a** were directly above point **b**. The later case would imply a wage difference between at any amount of labor demanded (where λ is the same) given by *eq*.(5). Because a higher level of p implies more buyers view this ability level as a perfect substitute, the demand for higher ability types shifts further to the right than implied by *eq.*(5). A larger wage difference between different levels of p means even more convexity in the wage with respect to p.

Figure One assumes an identical supply of individuals at different levels of p. There, as discussed in the previous paragraph, market demand effects cause even more convexity of the wage than is implied by looking at an individual buyer. Since a larger p implies one is closer in ability to a superstar, and the latter are presumably very scarce, it is plausible there are fewer non-superstars as p increases. If we still have a fixed number of individuals at each level of p, then market wages are determined as shown in *Figure Two*. The larger supply of those with $p = p_1$ implies more of them employed than those with $p = p_2$. Then, in equilibrium, the wage for those with p_1 would involve a higher level of λ than would equilibrium for those with p_2,

Figure One. The Market for Non-superstars.
Identical number at each level of *p*.

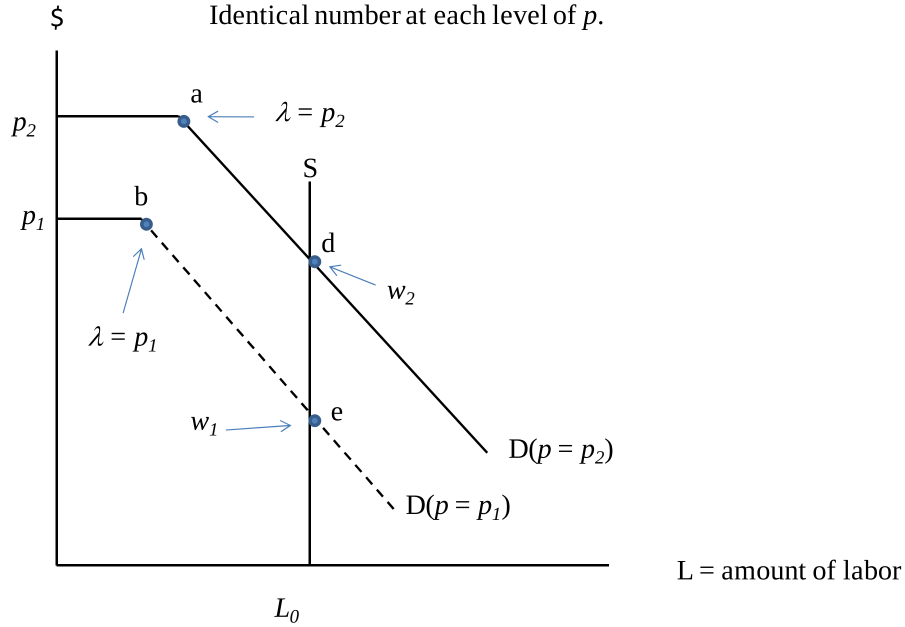
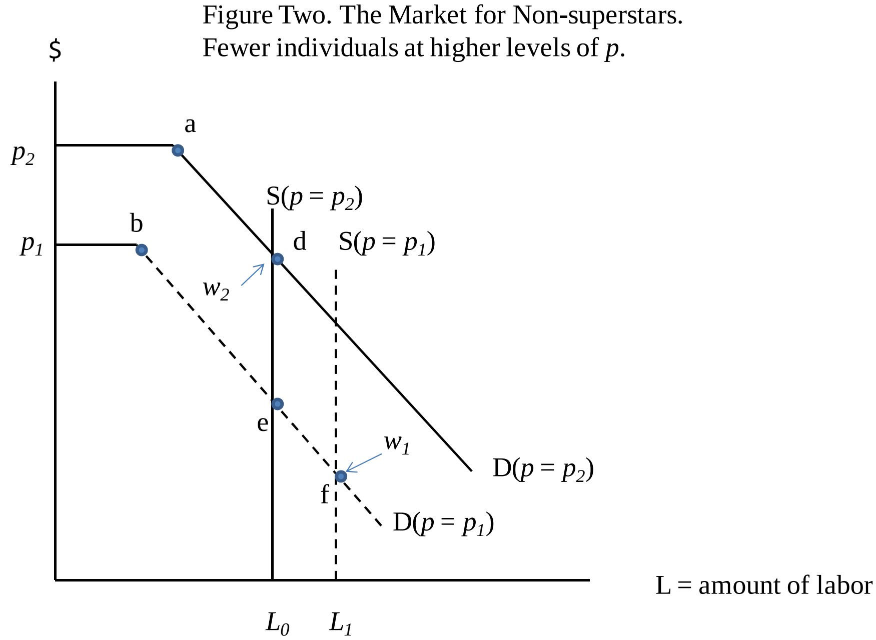


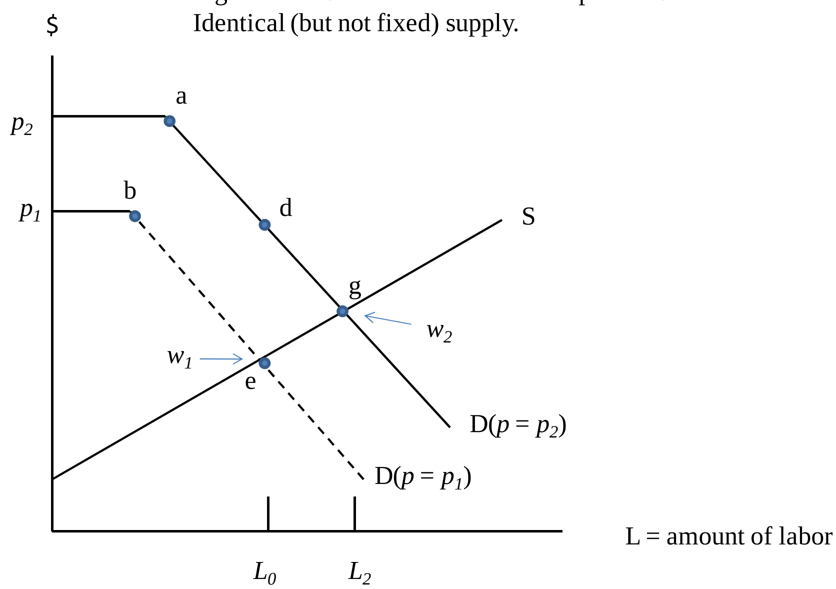
Figure Two. The Market for Non-superstars.
Fewer individuals at higher levels of *p*.



reflecting the value of λ for the L_1th individual. Since $\frac{\partial w}{\partial \lambda} < 0$, the convexity of *w* in *p* is even larger in this case than suggested previously and shown in *Figure One*.

If we have an identical, upward-sloping supply of individuals at any *p* (*Figure Three*), then the impact of *p* on *w* is attenuated. More employed at higher levels of *p*, implying larger levels of λ in equilibrium as *p* increases, which lowers *w* and the slope and convexity of *w* with respect to *p*.

Figure Three. The Market for Non-superstars.
Identical (but not fixed) supply.



6. Summary

The seminal paper by Rosen (1981) well explains several phenomena in superstar markets: total individual or firm earnings increasing with talent at an increasing rate, and market output and revenue highly concentrated among a few sellers. These results depend on superstars producing a much higher output than that for non-superstars. However, significantly larger output is not always optimal for superstars due to rising marginal cost. Yet some superstars have much higher per unit compensation than others, with such compensation apparently convex in talent.

The model herein explains convexity in wage rates due to imperfect substitutability between non-superstars and superstars when production is not certain. A virtue of the model is that it allows for a natural measure of the degree of substitutability between superstars and those with lesser talent. Consumers decide how close to the rate of success of a superstar a *set of non-superstars* must be in order for the combined effort of the latter to be essentially perfect substitutes for a superstar--- although *individual non-superstars* are imperfect substitutes.

One possible source of imperfect substitutability involves costs to consumers from delay when hiring non-superstars implies repeated attempts to produce the desired service. Although delay costs could be substantial, they are neither necessary nor sufficient for convexity in the wage with respect to talent.

With λ the success rate for a set of non-superstars relative to that for a superstar, what is not determined in the model herein is *why* λ might be relatively high (close to one, implying most sellers have very poor substitutes for superstars) or low (close to the probability of success of one non-superstar, implying virtually perfect substitutes). This topic is left for future research.

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